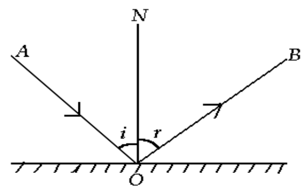
# GEOMETRICAL OPTICS

## *REFLECTION OF LIGHT*.

**1.0 Reflection of light on plane surfaces.**
This is the sending back of light rays by a reflecting surface

**1.1 Laws of reflection of light**.
Consider a ray of light AO incident on a plane surface and then reflected along OB as shown.

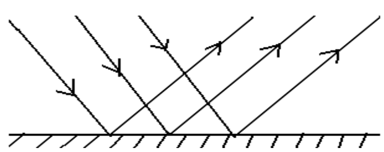



O = point of Incidence.
AO = incident ray
OB = reflected ray.
ON = normal to the reflecting surface
$\angle i$ = angle of incidence
$\angle r$ = angle of reflection

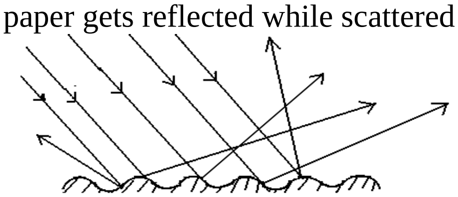
**LAW 1**:
The incident ray, the reflected ray, and the normal at the point of incidence all lie in the same plane.

**LAW 2**:
The angle of incidence is equal to the angle of reflection.

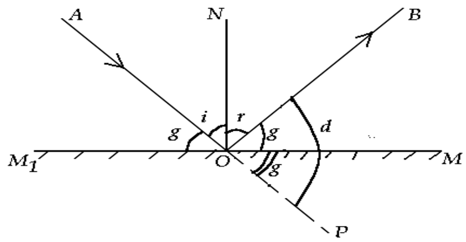
**1.2 Types of reflection**:
**(i)**Regular reflection: This occurs when a parallel beam of light incident on a smooth surface such as a plane mirror gets reflected as a parallel beam as shown.



**(ii)**Diffuse/Irregular reflection: This occurs when a parallel beam of light incident on a rough surface such as a paper gets reflected while scattered in different directions as shown



**1.3 Deviation of light at plane surfaces**
This is the property of a plane mirror to deviate incident light from its original direction to another.
Let g be the glancing angle made by the ray AO with the mirror $M_1M$ as shown
.




Since $\angle i = \angle r$, it follows that $\angle AOM_1 = \angle BOM$ From geometry, $\angle AOM_1 = \angle POM$ (vertically opposite angles)

By reflection, light is deviated from the original direction, AO, to a direction OB

Therefore the angle of deviation d = $\angle POB$

$$= \angle POM + \angle MOB$$

$$= g + g$$

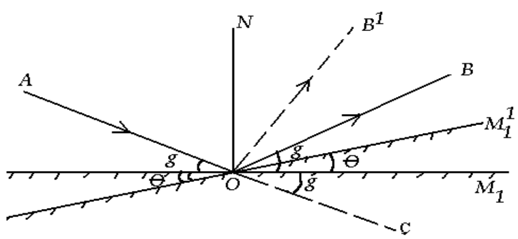
$$\therefore \quad \mathbf{d = 2g}$$

**Hence, the angle of deviation of a ray by a plane surface is twice the glancing angle**

### DEVIATION OF REFLECTED RAY BY ROTATED MIRROR

Let $M_1$ be the initial position of the mirror with ray AO making a glancing angle g.

By keeping the direction of the incident ray fixed, the mirror is rotated through an angle θ to a new position $M^1{}_1$ as shown.




Before mirror $M_1$ is rotated, the glancing angle is, g ,and the reflected ray is , OB ,

$\therefore$ Deviation by $M_1 = \angle COB = 2g$

For mirror $M^1{}_1$, the glancing angle is ( g + θ ) and the reflected ray is now $OB^1$

$\therefore$ Deviation by $M^1{}_1 = \angle COB^1 = 2 ( g + \theta )$

**Hence rotation of the reflected ray** $= \angle BOB^1$

$$= \angle COB^1 - \angle COB$$

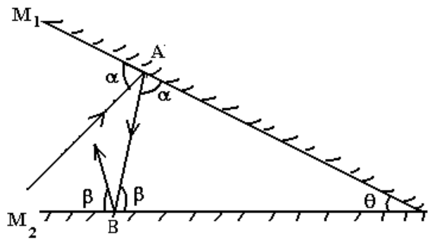
$$= 2 ( g + \theta ) - 2g$$

$$= \mathbf{2\theta}$$

**Thus, if the direction of an incident ray is constant, the angle of rotation of the reflected ray is twice the angle of rotation of the mirror.**

### DEVIATION BY SUCCESSIVE REFLECTIONS AT TWO INCLINED MIRRORS

Consider an incident ray of light reflected successively from two mirrors inclined at an angle θ to each other as shown



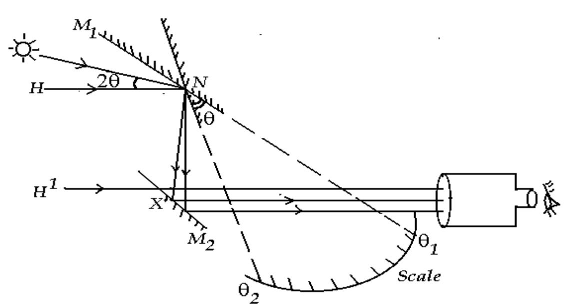

Let the glancing angles at A and B be $\alpha$ and $\beta$ respectively.
Deviation by $M_1$ = $2\alpha$ ( clockwise direction )
Deviation by $M_2$ = $2\beta$ ( clockwise direction )
Total deviation = $2\alpha + 2\beta$
= $2(\alpha + \beta)$ ----------------------(i)
More over, $\alpha + \beta + \theta = 180°$ ( Angle sum of a triangle )
$\Rightarrow$ $\alpha + \beta = (180° - \theta)$ ---------------------(ii)
Combining equation ( i ) and (ii) gives **Total deviation** = $2(180° - \theta)$ ( clockwise direction )
= $360° - 2\theta$ (clockwise direction)
**= $2\theta$ (anti-clockwise direction)**

**Thus, the deviation produced by two inclined mirrors is twice the angle between the mirrors when a ray under goes two successive reflections**.

**NOTE**
**(i)** Clock wise deviation **($360° - 2\theta$ )** **+** anti-clockwise deviation **($2\theta$)** **=** **$360°$**
**(ii)** The above result finds application in the sextant, a device for measuring the angle of elevation of the sun or stars.

**PRINCIPLE OF THE SEXTANT**:

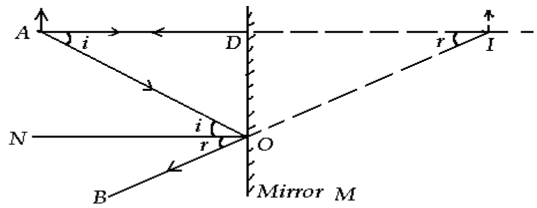
A sextant is an instrument used in navigation for measuring the angle elevation of the sun or stars.




A sextant consists of a fully silvered mirror $M_1$which can be rotated about a horizontal axis and a half silvered mirror $M_2$. Mirror $M_1$ is adjusted to become parallel to $M_2$ by rotating it until the image of the horizon $H^1$seen directly through the un silvered part of $M_2$ coincide with the image of the horizon H seen after successive reflections at N and X respectively. This mirror position $\theta_1$on the scale is noted. Mirror $M_1$ is again rotated in to a position such that the image of the star coincides with that of the horizon $H^1$. The new mirror position $\theta_2$ is again noted. The angle of rotation of mirror $M_1$ is $\theta = |\theta_2 - \theta_1|$. Thus the angle of rotation of the reflected ray is $2\theta$ and this is equal to the angle of elevation of the star.

### IMAGE FORMATION IN PLANE MIRRORS

Consider an object A placed in front of a mirror **M**.




A ray AD from A incident normally on the mirror at D is reflected back along DA. Thus this reflected ray appears to come from a point **I** behind the mirror. The intersection **I** of the rays AD and BO is the image position**.**

From above,

$$\angle DAO = \angle AON \text{ ---------------------------- (alternating angles)}$$
$$\angle AON = \angle NOB \text{ -------------------------- (2}^{nd}\text{ law of reflection)}$$
$$\angle NOB = \angle DIO \text{ ---------------------------- (corresponding angles)}$$

Combining all the equations gives

$$\angle DAO = \angle AON = \angle NOB = \angle DIO$$

$$\Rightarrow \quad \angle DAO = \angle DIO$$

$$\tan \angle DAO = \tan \angle DIO$$

$$\therefore \quad \frac{DO}{AD} = \frac{DO}{ID}$$

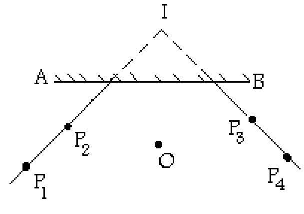
**Thus** $\quad$ **AD = ID**.

$\therefore$ T**he image is as far behind the mirror as the object is in front**

**CHARACTERISTICS OF IMAGES FORMED BY PLANE MIRRORS**

- It is virtual
- It is erect
- It is laterally inverted
- It is of the same size as the object
- It is as far behind the mirror as the object is in front

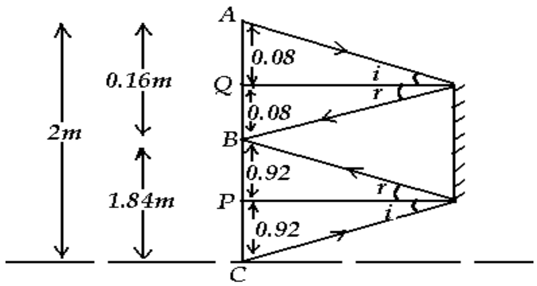
### IMAGE LOCATION IN PLANE MIRRORS




An object pin O is placed in front of a plane mirror AB, on a white sheet of paper. Looking from side A of the mirror, two pins $p_1$ and $p_2$ are placed so that they look to be in line with the image of the pin O. The experiment is repeated with pins $p_3$ and $p_4$ on side B. the pins and the mirror are removed and lines drawn through the pin marks $p_1p_2$ and $p_3p_4$ to meet at I. I is the position of the image.

**Minimum vertical length of a plane mirror**

**1**. A man 2m tall whose eye level is 1·84m above the ground looks at his image in a Vertical mirror. What must be the minimum vertical length of the mirror so that the man can see the whole of himself completely in the mirror ?




Rays from the top of the man are reflected from the top of the mirror and are incident in the man's eyes (point **B** is the man's eye level)

Since AQ = BQ then, $BQ = \frac{1}{2} \times 0{\cdot}16 = 0{\cdot}08$m

Similarly BP = PC. Thus $BP = \frac{1}{2} \times 1{\cdot}84 = 0{\cdot}92$m

$\therefore$ **The minimum length of the mirror** = BQ + BP

= 0·08 + 0·92

= **1m**

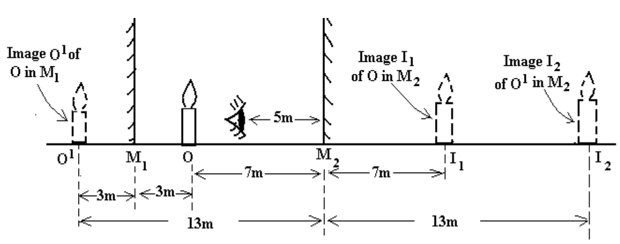
**Hence the minimum length of the mirror is half the height of the object**

**Formation of images by two plane mirrors facing each other**

Two vertical plane mirrors $M_1$ and $M_2$ are placed facing each other 10m apart in a room. A candle is set 3m from $M_1$ .An observer in the middle of the room looks in to mirror $M_2$ and sees two distinct images of the candle. How far are these images from the observer?

<u>ANALYSIS</u>

**T**he image formed in a plane mirror is as far behind the mirror as the object is in front




Consider the action of plane mirror $M_2$

Behind $M_2$, image distance due to O **= 7m**

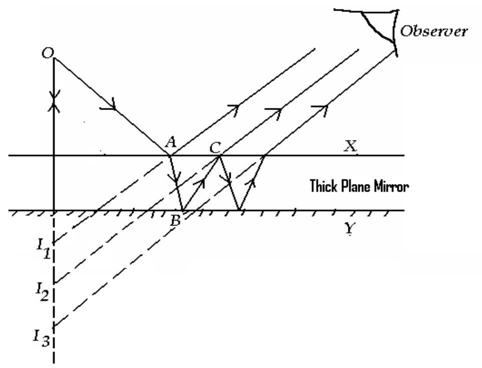
$\Rightarrow$ From the observer, image distance due to O **= (5 + 7) m = 12m**

Behind $M_2$, image distance due to $O^1$ **= 13m**

⇒ From the observer, image distance due to $O^1$ **= (5 + 13) m = 18m**

**FORMATION OF MULTIPLE IMAGES IN THICK PLANE MIRRORS**

When an object O is held in front of a thick plane mirror, a series of faint images are observed as shown




A thick plane has two plane surfaces say **,X,** and **,Y,**. At A, light under goes both reflection and transmission. The reflected light leads to the formation of image $\mathbf{I_1}$ and the transmitted light under goes reflection at B

At C, light undergoes both reflection and refraction. The refracted light leads to the formation of image $\mathbf{I_2}$. These successive total internal reflections bring about the formation of multiple images.

**NOTE**: **(i)**
Plane mirrors absorb some of the incident light at each reflection and thus forming less bright images.

**(ii)** The disadvantages of using plane mirrors as reflectors of light in optical instruments such as submarine periscopes are overcome by using reflecting prisms.

**COMPARISION OF PLANE MIRRORS AND REFLECTING PRISMS.**

**(i)** Un like in prisms, plane mirrors produce multiple images

**(ii)** The silvering in plane mirrors wears out with time while no silvering is required in prisms

**(iii)**Unlike in prisms, plane mirrors exercise loss of brightness when reflection occurs at its surface.

## EXERCISE

**1**. What is meant by reflection of light?

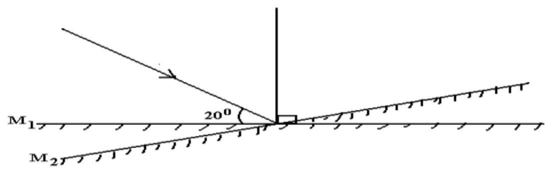
**2**. State the laws of reflection of light

**3**. Distinguish between regular and diffuse reflection of light

**4**. Show with the aid of a ray diagram that when a ray of light is incident on a plane mirror, the angle of deviation of a ray by the plane surface is twice the glancing angle.

**5**. Derive the relation between the angle of rotation of a plane mirror and the angle of deflection of a reflected ray, when the direction of the incident ray is constant.

**6.** An incident ray of light makes an angle of **20°** with the plane mirror in position $m_1$, as shown below




Calculate the angle of reflection, if the mirror is rotated through **6°** to position $m_2$ while the direction of the incident ray remains the same.

**7**. **(i)** Show that an incident ray of light reflected successively from two mirrors inclined at an angle $\theta$ to each other is deviated through an angle **2θ**.

**(ii)** Name one application of the result in **7(i)** above.

**8**. Describe how a sextant is used to determine the angle of elevation of a star.

**9.** Show that the image formed in a plane mirror is as far behind the mirror as the object is in front

**10**. State the characteristics of images formed by plane mirrors.

**11**.(i) What is meant by **No parallax method** as applied to location of an image**?**

**(ii)** Describe how the position of an image in a plane mirror can be located

**12**. Show that for a man of height, **H**, standing upright the minimum length of a vertical plane mirror in which he can see the whole of him self completely is $\frac{1}{2}$**H.**

**13** With the aid of a ray diagram, explain how a thick plane mirror forms multiple images of an object.

**14**. Give three reasons for using prisms rather than plane mirrors in reflecting optical instruments.

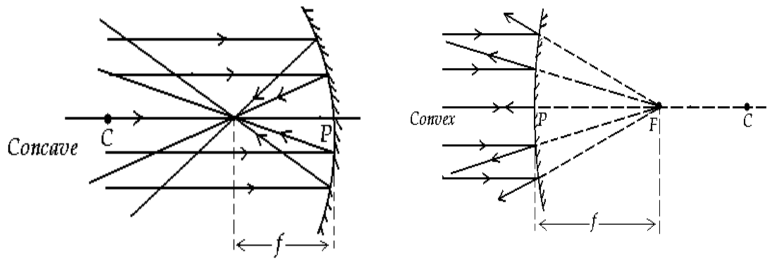
**REFLECTION AT CURVED MIRRORS**

Curved mirrors are of two types namely;

**(i)** **Concave (Converging) mirror:** it is part of the sphere whose centre **C** is in front of its reflecting surface.

**(ii) Convex (Diverging) mirror:** it is part of the sphere whose centre **C** is behind its reflecting surface.

Consider the reflection of a parallel narrow beam of light at curved mirrors as shown.




**Definitions**

1. **Centre of curvature C:** it is the centre of the sphere of which the mirror forms part.
2. **Radius of curvature r:** it is the radius of the sphere of which the mirror forms part.
3. **Pole of the mirror:** it is the mid-point (centre) of the mirror surface.
4. **Princpal axis CP**: it is the line that passes through the centre of curvature and the pole of the mirror.
5. **Secondary axes:** These are lines parallel to the principal axis of the mirror.
6. **Paraxial rays:** These are rays close to the principal axis and make small angles with the mirror axis.
7. **Marginal rays:** These are rays furthest from the principal axis of the mirror.
8. **(i) Principal focus "F" of a concave mirror:** it is a point on the principal axis where paraxial rays incident on the mirror and parallel to the principal axis converge after reflection by the mirror.

   **(ii). Principal focus "F" of a convex mirror:** it is a point on the principal axis where paraxial rays incident on the mirror and parallel to the principal axis appear to diverge from after reflection by the mirror

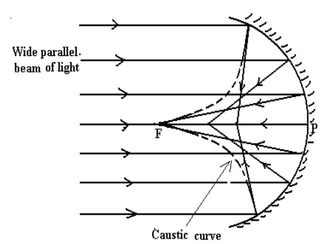
9. **(i) Focal length "f" of a concave mirror:** it is the distance from the pole of the mirror to the point where paraxial rays incident and parallel to the principal axis converge after reflection by the mirror.

   **(ii) Focal length "f" of a convex mirror:** it is the distance from the pole of the mirror to the point where paraxial rays incident and parallel to the principal axis appear to diverge from after reflection by the mirror.

10. **Aperture of the mirror:** it is the length of the mirror surface.

### REFLECTION OF A PARALLEL WIDE BEAM OF LIGHT AT CURVED MIRRORS

Consider the reflection of a wide parallel beam of light incident on a concave mirror as shown.



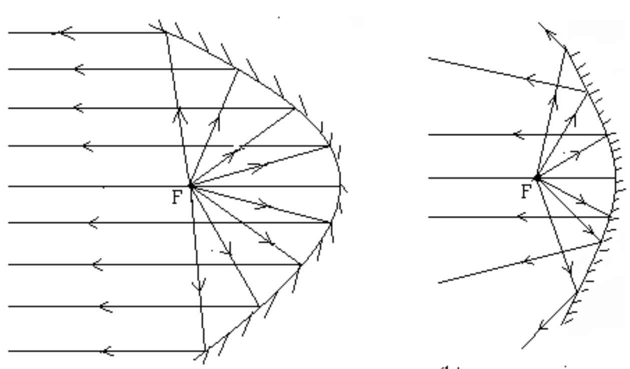

When a wide parallel beam of light is incident on a concave mirror, the different reflected rays are converged to different points. However these reflected rays appear to touch a surface known as a **caustic surface** having a cusp (an apex) at the principal focus **F**.

**NOTE**

**(i)** The marginal rays furthest from the principal axis are converged nearer to the pole of the mirror than the paraxial rays.

**(ii)** Similarly, if a wide parallel beam of light is incident on a convex mirror, the different reflected rays appear to have diverged from different points.

### COMPARISON OF CONCAVE AND PARABOLIC MIRRORS




When a lamp is placed at the principal focus of a concave mirror, only rays from this lamp that strike the mirror at points close to the principle axis will be reflected parallel to the principle axis and those striking at points well away from the principal axis will be reflected in different directions and not as a parallel beam as seen in (b) above. In this case the intensity of the reflected beam practically diminishes as the distance from the mirror increases.

When a lamp is placed at the principal focus of a parabolic mirror, all rays from this lamp that strike the mirror at points close to and far from the principle axis will be reflected parallel to the principle axis as seen in (a) above. In this case the intensity of the reflected beam remains practically undiminished as the distance from the mirror increases. This accounts for the use of parabolic mirrors as search lights other than concave mirrors.

**GEOMETRICAL RULES FOR THE CONSTRUCTION OF RAY DAIGRAMS**

The following is a set of rules for easy location of the images formed by spherical mirrors

1. Rays parallel to the principal axis are reflected through the principal focus.
2. Rays through the principal focus are reflected parallel to the principal axis.
3. Rays passing through the centre of curvature are reflected back along their own paths.
4. Rays incident to the pole are reflected back, making the same angle with the principal axis.

**NOTE**:

**(i)** The normal due to reflection at the mirror surface at any point must pass through the centre of curvature.

**(ii)** The image position can be located by the intersection of two reflected rays initially coming from the object.

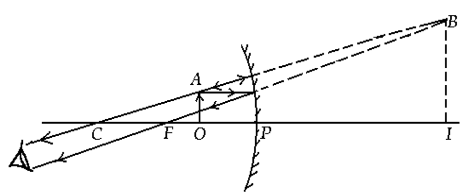
**REAL AND VIRTUAL IMAGES**

**A REAL IMAGE**: This is the image formed by the actual intersection of light rays from an object and can be received on the screen.

**A VIRTUAL IMAGE**: This is the image formed by the apparent intersection of light rays and can not be received on the screen

**IMAGES FORMED BY A CONCAVE MIRROR**

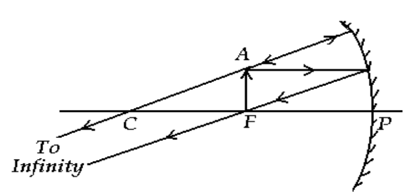
The nature of the image formed by a concave mirror is either real or virtual depending on the object distance from the mirror as shown below;




**Object between F and P the image is**

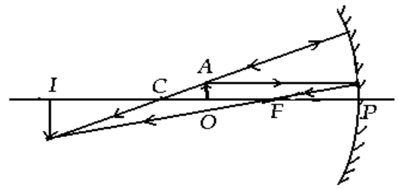
1) Behind the mirror
2) Virtual
3) Erect
4) Magnified

The property of a concave mirror to form erect, virtual and a magnified image when the object is nearer to the mirror than its focus makes it useful as a shaving mirror and also used by dentists for teeth examination.

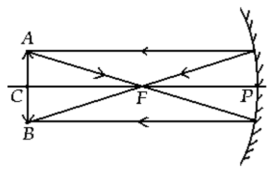



**Object at F the image is**

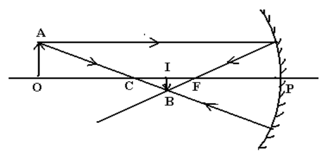
1) at infinity



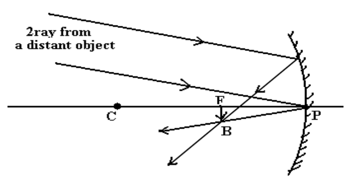

**Object between F and C the image is**
1) Beyond C
2) Real
3) Inverted
4) Magnified




**Object at C the image is**
1) At C
2) Real
3) Inverted
4)Same size as the object




**Object beyond C the Image is**
1) Between C and F
2) Real
3) Inverted
4) Diminished




**Object at infinity the image is**
1) At F
2) Real
3) Inverted
4) Diminished

**NOTE;**

Generally the image of an object in a concave mirror is virtual only when the object is nearer to the mirror than its focus.

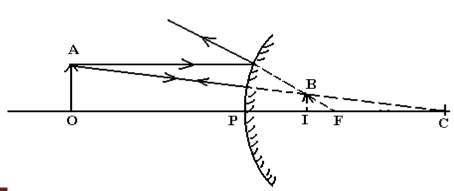
**USES OF CONCAVE MIRRORS**

(i) They are used as shaving mirrors.
(ii) They are used by dentists for teeth examination.
(iii)They are used as solar concentrators in solar panels.
(iv)They are used in reflecting telescopes, a device for viewing distant objects
(v) They are used in projectors, a device for showing slides on a screen.

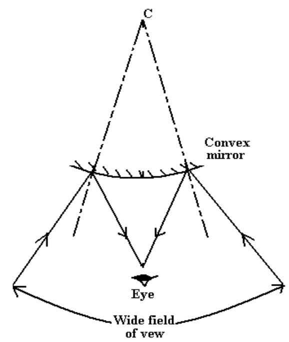
**IMAGES FORMED BY A CONVEX MIRROR**

The image of an object in a convex mirror is erect, virtual, and diminished in size no matter where the object is situated as shown below

.



In addition to providing an erect image, convex mirrors have got a wide field of view as illustrated below.



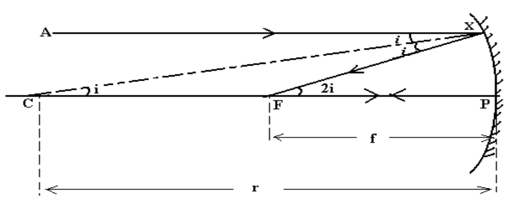

**USES OF CONVEX MIRRORS**

(i)They are used as car driving mirror

(ii)They are used in reflecting telescopes, a device for viewing distant objects

**RELATIONSHIP BETWEEN THE FOCAL LENGTH f AND THE RADIUS OF CURVATURE r OF A CURVED MIRROR.**

Consider the reflection of a paraxial ray, AX, parallel to the principal axis of a concave mirror as shown.




Taking **FP** = focal length and **C** = centre of curvature, then ,**CX**, is the normal to the mirror surface and, **CP**, is the radius of curvature.

If, X, is close to P, then $\tan 2i \approx 2i = \frac{XP}{FP}$ ---------------- (for small angles)

$\Rightarrow \quad 2i = \frac{XP}{FP}$ -----------------------------------------------(i)

Similarly $\tan i \approx i = \frac{XP}{CP}$

$\Rightarrow \quad i = \frac{XP}{CP}$ ----------------------------------------------- (ii)

Combining equations (i) and (ii) gives,

$$2\frac{XP}{CP} = \frac{XP}{FP}$$

Canceling XP throughout and simplifying for CP gives

CP = 2FP where CP = r, FP = f

$\therefore$ **r = 2f**.

**Thus, the radius of curvature of a concave mirror is twice its focal length.**

**NOTE:**

It can be shown that the relation between f and r holds for both concave and convex mirrors**.(This is left as an exercise for the reader)**

**MIRROR FORMULA AND SIGN CONVENTION**

In order to obtain a formula which holds for both concave and convex mirrors, a Sign rule or convention must be obeyed and the following shall be adopted.

**(i)** Distances of real objects and images are positive.

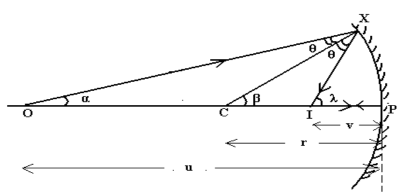
**(ii)** Distances of virtual objects and images are negative.

**NOTE:**

A concave mirror has a positive focal length while a convex mirror has a negative focal length.

**CONCAVE (CONVERGING) MIRROR FORMULA**

Consider the incidence of ray **OX** on to a concave mirror from a point object **O** placed along the principal axis and then suddenly reflected in the direction XI making an angle $\theta$ with the normal CX.




Ray OP strikes the mirror incident normally at P and thus reflected back along its own path. The point of intersection **I** of the two reflected rays is the image position.

From $\Delta$ OXC, $\alpha + \theta = \beta$ -------------------------------(i)

From $\Delta$ CXI, $\beta + \theta = \lambda$ ------------------------------(ii)

Equation (i) $-$ Equation (ii) eliminates $\theta$ to give.

$\alpha - \beta = \beta - \lambda$

$\Rightarrow \alpha + \lambda = 2\beta$ -----------------------------------------(a)

if X is very close to P, then

$$\alpha \approx \tan\alpha = \frac{XP}{u}, \quad \beta \approx \tan\beta = \frac{XP}{r}, \quad \text{and} \quad \lambda \approx \tan\lambda = \frac{XP}{v}$$

Equation (a) now becomes

$$\frac{XP}{u} + \frac{XP}{v} = \frac{2XP}{r}$$

$$\Rightarrow \quad \frac{1}{u} + \frac{1}{v} = \frac{2}{r}$$

**But** $r = 2f$

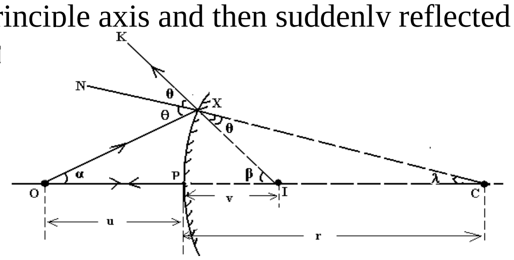
**Hence** $$\mathbf{\frac{1}{u} + \frac{1}{v} = \frac{1}{f}}$$

**NOTE:**

The focal length of a concave mirror is a positive distance.

**CONVEX (DIVERGING) MIRROR FORMULA**:

Consider the incidence of ray **OX** on to a convex mirror from a point object **O** placed along the principle axis and then suddenly reflected in the direction **XK** making an angle θ with the normal **X**




The incident rays **OX** and **OP** after reflection appears to have come from point **I** behind the mirror which is the position of the virtual image.

From $\Delta$ XIC, $\theta + \lambda = \beta$

$\Rightarrow \theta = \beta - \lambda$ --------------------------------(i)

From $\Delta$ OXI, $\alpha + \beta = 2\theta$ ---------------------------------(ii)

Substituting equation (i) into (ii) gives.

$\alpha + \beta = 2(\beta - \alpha)$

$\Rightarrow \alpha - \beta = -2\lambda$ ---------------------------------------(a)

If X is very close to P, then

$\alpha \approx \tan\alpha = \frac{XP}{u}$, $\beta \approx \tan\beta = \frac{XP}{-v}$, (I is virtual) and $\lambda \approx \tan\lambda = \frac{XP}{-r}$ (C is virtual).

Equation (a) now becomes

$$\frac{XP}{u} - \frac{XP}{-v} = \frac{-2XP}{-r}$$

$$\Rightarrow \quad \frac{1}{u} + \frac{1}{v} = \frac{2}{r}$$

**But** $r = 2f$

**Hence** $\mathbf{\frac{1}{u} + \frac{1}{v} = \frac{1}{f}}$

**NOTE:**

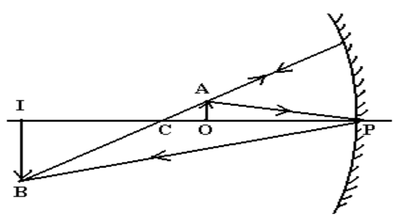
The focal length of a convex mirror is a negative distance.

**FORMULA FOR MAGNIFICATION**

Linear magnification, m, = $\frac{\text{Image height}}{\text{Object height}} = \frac{\text{Image distance}}{\text{Object distance}}$.

**PROOF**

Consider the incidence of ray **AP** on to the pole of a concave mirror from an object of height **h** placed a distance, **u,** from the mirror and then reflected back making the same angle with the principal axis to form an image of height $\mathbf{h_1}$, located at distance, **v**, from the mirror as shown




ray **AP** makes an angle $\theta$ with the normal **OP**, then,

From $\Delta$ OAP, $\tan\theta = \frac{h}{u}$ --------------------(i)

From $\Delta$ IPB, $\tan\theta = \frac{h_1}{v}$ ----------------------------(ii)

Equating equation (i) and (ii) gives.

$$\frac{v}{u} = \frac{h_1}{h}$$

**Thus magnification, m,** $= \mathbf{\frac{v}{u} = \frac{h_1}{h}}$

**NOTE:**

**(i)** No signs need be inserted in the magnification formula. **(ii)** Using the mirror formula, a connection relating magnification to the focal length of the mirror with either the object distance or the image distance can be established.

**RELATIONSHIP CONNECTING m, v and f**

Using the mirror formula $\frac{1}{} = \frac{1}{} + \frac{1}{}$

f u v

Multiplying, v, throughout the expression gives,

$$\frac{v}{f} = \frac{v}{u} + 1$$

**But** $\frac{v}{u} = m$

-

$$\Rightarrow \frac{v}{f} = m + 1$$

$$\therefore \mathbf{m = \frac{v}{f} - 1}$$

**RELATIONSHIP CONNECTING m, u and f**

Using the mirror formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Multiplying, u, throughout the expression gives,

$$\frac{u}{f} = 1 + \frac{u}{v}$$

**But** $\frac{u}{v} = \frac{1}{m}$

$$\Rightarrow \frac{u}{f} = 1 + \frac{1}{m}$$

$$\therefore \mathbf{\frac{1}{m} = \frac{u}{f} - 1}$$

**EXAMPLES** **(1)**

An object is placed 10cm in front of a concave mirror of focal length 15cm. Find the image position and magnification.

**Solution**:

For a concave mirror, focal length f = +15cm, u = 10cm.

Using the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ gives

$$v = \frac{f \times u}{u - f} = \frac{15 \times 10}{10 - 15} = \mathbf{-30cm}.$$

The negative sign implies that the image formed is **virtual** and it is formed 30cm from the mirror.

$\therefore$ **Magnification, m** $= \frac{v}{u} = \frac{30}{10} = \mathbf{3}$

**(2)** The image of an object in a convex mirror is 6cm from the mirror. If the radius of curvature of the mirror is 20cm, find the object position and the magnification.

**Solution:**

For a convex mirror, $f = -\frac{r}{2} = -\frac{20}{2} = -10cm$

$v = -6cm$ (The image in a convex mirror is always virtual)

Using the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ gives

$$u = \frac{f \times v}{v - f} = \frac{-10 \times -6}{-6 - (-10)} = \mathbf{15cm}$$

Magnification, $m = \frac{v}{u} = \frac{6}{15} = \mathbf{0{\cdot}4}$

**3.** Show that an object and its image coincide in position at the centre of curvature of a concave mirror. Hence find the magnification produced in this case.

**Solution**

At the centre of curvature of a concave mirror, object distance **u = r** where r is the radius of curvature of the mirror.

Using the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$ gives

$$\mathbf{v = \frac{r \cdot u}{2u - r}} = \frac{r \cdot r}{2r - r} = \frac{r^2}{r} = \mathbf{r}$$

Thus the image is also formed at the centre of curvature and therefore it coincides in position with its object.

**Hence,** Magnification, $\mathbf{m} = \frac{\mathbf{v}}{\mathbf{u}} = \frac{\mathbf{r}}{\mathbf{r}} = \mathbf{1}$

$\therefore$ **The object and its image are of the same size in this case.**

**4**. A concave mirror forms on a screen a real image of three times the size of the object. The object and screen are then moved until the image is five times the size of the object. If the shift of the screen is **30cm,** determine the

**(i**) focal length of the mirror

**(ii)** shift of the object

**Solution.**

**(i)** Using the relation $m = \frac{v}{f} - 1$ for values of $m_1 = 3$ and $m_2 = 5$ gives

$$v_1 = (m_1 + 1)f \text{ ----------------------(i)}$$
$$v_2 = (m_2 + 1)f \text{ ----------------------(ii)}$$

For $\mathbf{m_2} > \mathbf{m_1}$, it follows that $\mathbf{v_2} > \mathbf{v_1}$

Thus, the given shift in the image $= v_2 - v_1 = 30cm$

Equation (**ii**) – Equation (**i**) gives

$$v_2 - v_1 = (m_2 - m_1)f$$

$$\Rightarrow \quad \mathbf{f} = \frac{\mathbf{v_2 - v_1}}{\mathbf{m_2 - m_1}} = \frac{30}{5 - 3} = \mathbf{15cm.}$$

**(ii)** Using the relation $\frac{1}{m} = \frac{u}{f} - 1$ for values of $m_1 = 3$ and $m_2 = 5$ gives

$$u_1 = \frac{f}{m_1} + 1 \text{ -------------------- (a)}$$

$$u_2 = \frac{f}{m_2} + 1 \text{ -------------------- (b)}$$

For $m_2 > m_1$, it follows that $u_1 > u_2$

Thus, the required shift in the object $= u_1 - u_2$

Equation (a) - Equation (b) gives

$$u_1 - u_2 = \left(\frac{1}{m_1} - \frac{1}{m_2}\right)f$$

$$\Rightarrow \quad u_1 - u_2 = \frac{\mathbf{(m_2 - m_1)f}}{\mathbf{m_2 \cdot m_1}} = \frac{15(5-3)}{5 \times 3} = \mathbf{2cm.}$$

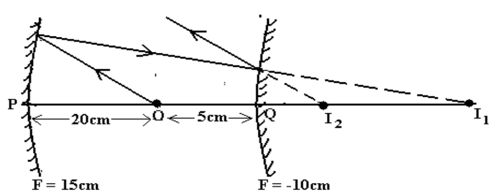
$\therefore$ The required shift of the object = **2cm**

**5.** A concave mirror **P** of focal length 15cm faces a convex mirror **Q** of focal length 10cm placed 25cm from it. An object is placed between **P** and **Q** at a point 20cm from **P**.

(i) Determine the distance from **Q** of the image formed by reflection**,** first in **P** and then in **Q**

(ii) Find the magnification of the image formed in **(i)** above

**Solution**




Consider the action of a concave mirror

$u = 20cm$, and $f = 15cm$

Using the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ gives

$$v = \frac{f \cdot u}{u - f} = \frac{20 \times 15}{20 - 15} = 60cm$$

$\therefore$ The image distance from a concave mirror = **60cm**

**Thus,** the image distance behind a convex mirror = **60 – (5 + 20)cm**

**= 35cm.**

Consider the action of a convex mirror

The image formed by a concave mirror acts as a virtual object for the convex mirror. Thus u = -35cm and f = -10cm

Using the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ gives

$$v = \frac{f \cdot u}{u - f} = \frac{-10 \times -35}{-35 - {}^{-}10} = \mathbf{-14cm}$$

∴ A final virtual image is **14cm** behind the convex mirror

**(**ii) Magnification, $\mathbf{m} = \frac{h_2}{h_o} = \frac{h_2}{h_1} \times \frac{h_1}{h_o} = \mathbf{m_2} \times \mathbf{m_1}$

$\therefore\ \mathbf{m} = \frac{\mathbf{v_1}}{\mathbf{u_1}} \times \frac{\mathbf{v_2}}{\mathbf{u_2}}$ where $u_1 = 20cm,\ v_1 = 60cm,\ u_2 = 35cm$ and $v_2 = 14cm$.6. A

**Thus** $\mathbf{m} = \frac{60}{20} \times \frac{14}{35} = \mathbf{1 \cdot 2}$

small convex mirror is placed **60cm** from the pole and on the axis of a large concave mirror of radius of curvature **200cm.** The position of the convex mirror is such that a real image of a distant object is formed in the plane of a hole drilled through the concave mirror at its pole.

**(a) (i)** Draw a ray diagram to show how a convex mirror forms an image of a non-axial point of a distant object

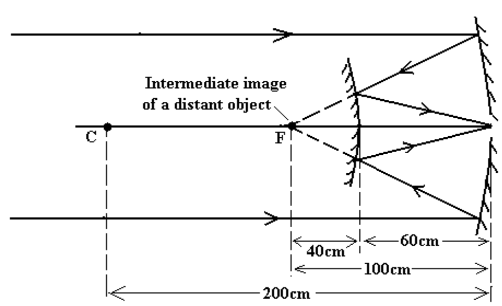
**(ii)** Suggest a practical application for the arrangement of the mirrors in **a (i)** above**.**

**(b)** Calculate the

**(i)** radius of curvature of the convex mirror.

**(ii)** height of the real image if the distant object subtends an angle of **0·5°** at the pole of the convex mirror.

**Solution(b) i)**




**(ii**) The mirror arrangement finds application in a reflecting telescope, a device for viewing distant objects

**(b) (i) Consider the action of a concave mirror**

The image of a distant object is formed at the principal focus of the concave mirror. This image acts as a virtual object for a convex mirror.

**Consider the action of a convex mirror**

u = -40cm and v = 60cm

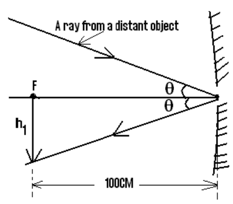
using the mirror formular $\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$ gives

$$r = \frac{2uv}{u+v} = \frac{2 \times -40 \times 60}{-40 + 60} = \mathbf{-240cm}$$

**The required radius of curvature r = 240cm**

**(ii) Consider the magnification produced by a convex mirror**

Let $\mathbf{h_1} =$ height of the intermediate image formed by a concave mirror as shown.




$$\text{from above, } \tan 0\cdot5^o = \frac{h_1}{100}$$

$$\Rightarrow h_1 = 100 \tan 0\cdot5^o = 0\cdot8727\text{cm}$$

let $h_2 =$ height of the image formed by a convex mirror

$$\text{magnification } M = \frac{h_2}{h_1} = \frac{v}{u}$$

$$\Rightarrow \mathbf{h_2} = \frac{v}{u} \times h_1 = \frac{60}{40} \times 0\cdot8727 = \mathbf{1\cdot3cm}$$

**Required image height = 1·3cm**

**EXERCISE**

**1**.Define the terms centre of curvature, radius of curvature, principal focus and focal length of a converging mirror.

**2**.Distinguish between real and virtual images.

**3.** Explain with the aid of a concave mirror the term **a caustic surface.**

.

**4**.Explain why a parabolic mirror is used in searchlights instead of a concave mirror

**5** An object is placed a distance **u** from a concave mirror. The mirror forms an image of the object at a distance **v.** Draw a ray diagram to show the path of light when the image formed is:

**(i) real**

**(ii) virtual**

**6**. Give two instances in each case where concave mirrors and convex mirrors are useful.

**7**. **(i)** Explain the suitability of a concave mirror as a shaving mirror.
**(ii)** Explain with the aid of a ray diagram why a convex mirror is used as a car driving mirror.

**8.** Show with the aid of a ray diagram, that the radius of curvature of a concave mirror is twice the focal length of the mirror

**9**. Use a geometrical ray diagram to derive the relation $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ for a concave mirror. **10.** Derive the relation connecting the radius of curvature **r** object distance **u** and image distance **v** of a diverging mirror.

**11.** An object is placed perpendicular to the principal axis of a concave mirror of focal length f at a distance **(f+ x)** and a real image of the object is formed at a distance **(f + y).** Show that the radius of curvature **r** of the mirror is given by $\mathbf{r = 2\sqrt{xy}}$

**12.** (i) Define the term **linear magnification.**

(ii) Show that in a concave mirror, **linear magnification** $= \dfrac{\textbf{image distance}}{\textbf{object distance}}$.

(iii) A concave mirror of focal length 15cm forms an erect image that is three times the size of the object. Determine the object and its corresponding image position.

(iv) A concave mirror of focal length 10cm forms an image five times the height of its object. Find the possible object and corresponding image positions.

**[Ans: (iii) u = 10cm, v = –30cm (iv) u = 12cm, v = 60cm OR u = 8cm, v =–40cm ]**

**13.** A concave mirror forms on a screen a real image which is twice the size of the object. The object and screen are then moved until the image is five times the size of the object. If the shift of the screen is 30cm, determine the

(**i**) focal length of the mirror
**(ii)** shift of the object

**[Answers: (i) f = 10cm (ii) 3cm ]**

**14.** A concave mirror of radius of curvature **20cm** faces a convex mirror of radius of curvature **10cm** and is **28cm** from it. If an object is placed midway between the mirrors, find the nature and position of the image formed by reflection first at the concave mirror and then at the convex mirror.

**[ Answer:** A final virtual image is **17·5cm** behind the convex mirror **]**

**15.** A small convex mirror is placed **100cm** from the pole and on the axis of a large concave mirror of radius of curvature **320cm.** The position of the convex mirror is such that a real image of a distant object is formed in the plane of a hole drilled through the concave mirror at its pole.

**(a) (i)** Draw a ray diagram to show how a convex mirror forms an image of a non-axial point of a distant object

**(ii)** Suggest a practical application for the arrangement of mirrors in **a (i)** above**.**

**(iii)** Calculate the radius of curvature of the convex mirror

**(b)** If the distant object subtends an angle of $\mathbf{3 \times 10^{-3}}$ **radians** at the pole of the concave mirror, calculate the

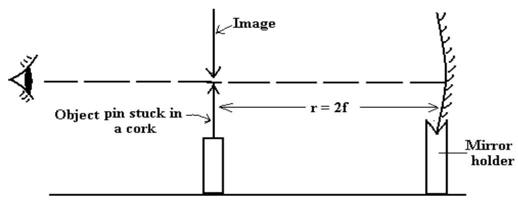
**(i)** size of the real image that would have been formed at the focus of the concave mirror.

**(ii)** size of the image formed by the convex mirror

**[ Ans: (a) (iii) 150cm (b) (i) 0·48cm (ii) 0·8cm ]**

**DETERMINATION OF THE FOCAL LENGTH OF A CONCAVE MIRROR.**

**<u>Method (1) Using a pin at C</u>**



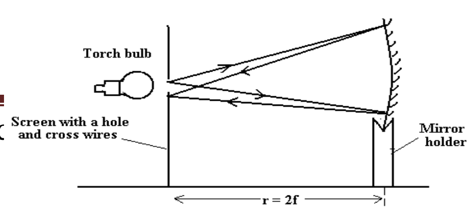

An object pin is placed in front of a mounted concave mirror so that its tip lies along the axis of the mirror. The position of the pin is adjusted until it coincides with its image such that there is no parallax between the pin and its image. The distance **r** of the pin from the mirror is measured. The required focal length $\mathbf{f = \frac{1}{2} r}$**.**

NOTE :

**(i)** In the position where there is no parallax between the object pin and its image, there is no relative motion between the object and its image when the observer moves the head from side to side.

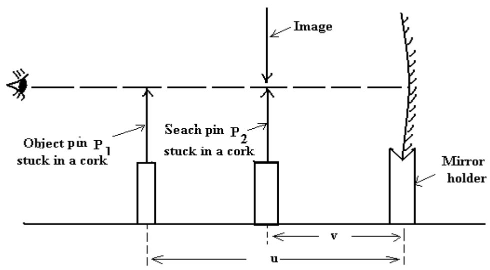
(ii) When the pin coincides with its image, the rays are incident normal to the mirror and are thus reflected along their own path. Therefore the pin coincides with its image at the centre of curvature of the mirror.

**<u>Method (2) Using an illuminated object at C</u>**



Align an illuminated bulb, a screen and a concave mirror mounted in a holder as shown above. The mirror position is adjusted to or from the screen until a sharp image of the cross-wire is formed on the screen besides the object. The distance **r** of the mirror from the screen is measured. The required focal length $\mathbf{f = \frac{1}{2} r}$.

**Method (3) Using no parallax method in locating v**



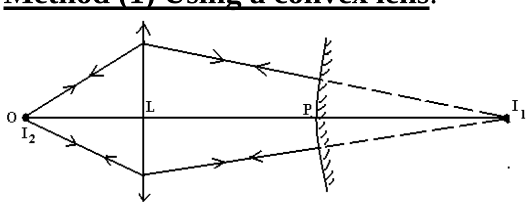

An object pin **P₁** is placed at a distance **u** in front of a mounted concave mirror so that its tip lies along the axis of the mirror. A search pin **P₂** placed between the mirror and pin **p₁** is adjusted until it coincides with the image of pin **p₁** by no-parallax method.
The distance **v** of pin **p₂** from the mirror is measured**.** The procedure is repeated for several values of **u** and the results are tabulated including values of **uv**, and **u+v.** A graph of **uv** against **u+v** is plotted and the slope **s** of such a graph is equal to the focal length **f** of the mirror.

**NOTE:**

If a graph of $\frac{1}{u}$ against $\frac{1}{v}$ is plotted, then each intercept **c** of such a graph is equal to $\frac{1}{f}$. **Thus** $\mathbf{f = \frac{1}{c}}$

**DETERMINATION OF FOCAL LENGTH OF A CONVEX MIRROR.**

**Method (1) Using a convex lens**.

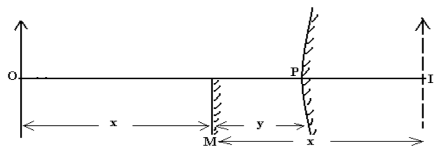



An object **O** is placed in front of a convex lens to form a real image on the screen at **I₁**. The distance **LI₁** of the screen from the lens is then measured. A convex mirror is placed between the lens and the screen. The mirror is then moved along the axis **$OI_1$** until an image **$I_2$** is formed besides **O**. The distance **LP** is measured. The required focal length $\mathbf{f = \frac{PI_1}{2}}$, where $PI_1 = LI_1 - LP$

**NOTE ;**

When the incident rays from an object are reflected back along the incident path, a real inverted image is formed besides the object in which case the rays strike the mirror normally. Therefore they will if produced pass through the centre of curvature of the mirror thus distance **$PI_1$ = radius of curvature**

**Method (2) Using No parallax.**




An object pin **O** is placed in front of a convex mirror. A virtual diminished image is formed at **I**. A plane mirror **M** is placed between **O** and **P** so as to intercept half the field of view of the convex mirror. Mirror **M** is adjusted until its own image of **O** coincides with **I** by no parallax method. Measure the distances **x** and **y.** The focal length of the mirror is then calculated from $\frac{\mathbf{1}}{\mathbf{f}} = \frac{\mathbf{1}}{\mathbf{u}} + \frac{\mathbf{1}}{\mathbf{v}}$ where $u = (x + y)$ and $v = -(x - y) = y - x$.

**Note:**

(i) The two images coincides when they are as far behind the plane mirror as the object is in front.

(ii) Substituting for u = x + y and v = y – x in the mirror formula gives $f = \frac{y^2 - x^2}{2y}$

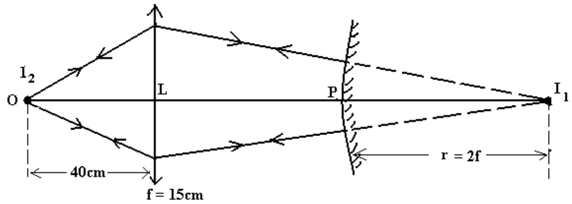
**EXAMPLES**

**1.** An object **O** is placed **40cm** in front of a convex lens of focal length **15cm** forming an image on the screen. A convex mirror situated **4cm** from the lens in the region between the lens and the screen forms the final image besides object **O**.

**(i )** Draw a ray diagram to show how the final image is formed.

**(ii)** Determine the focal length of the convex mirror.

**Solution**




Consider the action of a convex lens

u = 40cm, and f = 15cm

Using the lens formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ gives

$$v = \frac{f \cdot u}{u - f} = \frac{15 \times 40}{40 - 15} = \mathbf{24cm}$$

The radius of curvature **r = (24 – 4)cm = 20cm**

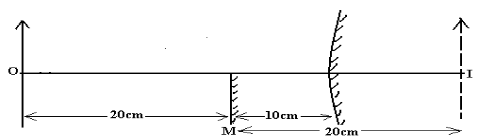
**Using the relation r = 2f**

$\Rightarrow$ $2f = 20cm$

$\therefore$ $f = 10cm$

**Thus f = -10cm** "The centre of curvature of a convex mirror is virtual"

4. A plane mirror is placed **10cm** in front of a convex mirror so that it covers about half of the mirror surface. A pin **20cm** in front of the plane mirror gives an image in it, which coincides with that of the pin in the convex mirror. Find the focal length of the convex mirror.




**Consider the action of a convex mirror**

u = 30cm and v = – (20 – 10 ) = –10cm "The image formed is virtual "

Using the mirror formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ gives

$$v = \frac{fu}{u - f} = \frac{-10 \times 30}{30 + -10} = -15cm$$

$\therefore$ $f = -15cm$

**EXERCISE**

**1**.Describe an experiment to determine the focal length of a concave mirror.

**2**.You are provided with the following pieces of apparatus: A screen with cross wires, a lamp, a concave mirror, and a meter ruler. Describe an experiment to determine the focal length of a concave mirror using the above apparatus.

**3**. Describe an experiment, including a graphical analysis of the results to determine the focal length of a concave mirror using a no parallax method.

**4.** Describe an experiment to measure the focal length of a convex mirror

**5**.Describe how the focal length of a diverging mirror can be determined using a convex lens.

**6**.Describe how the focal length of a convex mirror can be obtained using a plane mirror and the no parallax method.

**7**. A plane mirror is placed at a distance **d** in front of a convex mirror of focal length **f** such that it covers about half of the mirror surface. A pin placed at a distance **L** in front of the plane mirror gives an image in it, which coincides with that of the pin in the convex mirror. With the aid of an illustration, Show that $\mathbf{2df = d^2 - L^2}$

**REFRACTION AT PLANE SURFACES**:

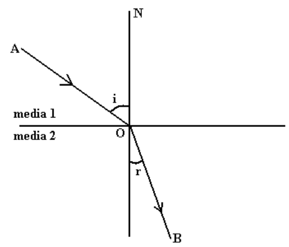
**Refraction** is the bending of light at an interface between two media of different optical densities

**NOTE ;**

Light is refracted because it has different speeds in different media.

**LAWS OF REFRACTION**

Consider a ray of light incident on an interface between two media as shown.




O = Point of incidence.
AO = Incident ray
OB = Refracted ray.
ON = Normal at **O**
$\angle i$ =Angle of incidence
$\angle r$ =Angle of refraction

**LAW 1:**

The incident ray, the refracted ray and the normal at the point of incidence all lie in the same plane.

**LAW 2:**

The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for a given pair of media.

Thus $\frac{\sin i}{\sin r}$ = **a constant (n).** This is also known as **Snell's law.**

**REFRACTIVE INDEX. [ n ]**

is the ratio of the sine of angle of incident to the sine of angle of refraction for a ray of light traveling from air in to a given medium.

**OR**

Is the ratio of the speed of light in a vacuum to speed of light in a medium.

Thus **Refractive index**, $n = \frac{\text{speed of light in a vacuum, } c}{\text{speed of light in a medium, } c_m}$

Where speed of light in a vacuum $c = 3{\cdot}0 \times 10^8\ ms^{-1}$.

**NOTE:**

The refractive index, **n** for a vacuum is **1**. However if light travels from air to another medium, the value of **n** is slightly greater than **1**.For example, **n = 1·33** for water and **n =1·5** for glass.

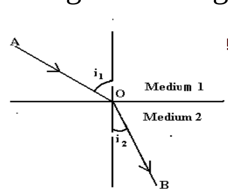
**THE PRINCIPLE OF REVERSIBILITY OF LIGHT.**

It states that the paths of light rays are reversible. This means that a ray of light can travel from medium **1** to **2** and from **2** to **1** along the same path.

**RELATIONS BETWEEN REFRACTIVE INDICES**

**CASE I:**

Consider a ray of light traveling from **medium 1 (air)** to **medium 2 (glass)** as shown.



Suppose $i_1$ and $i_2$ are the angles of incidence and refraction respectively in **medium 1** and **medium 2,** then

For light traveling from **(1)** to **(2)**, refractive index $_1n_2 = \dfrac{\sin i_1}{\sin i_2}$ ------------------**(i)**

For light traveling from **(2)** to **(1)**, refractive index $_2n_1 = \dfrac{\sin i_2}{\sin i_1}$

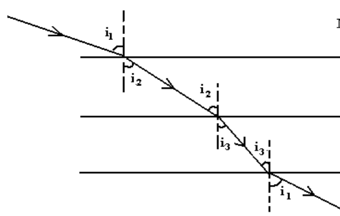
$\Rightarrow \dfrac{1}{_2n_1} = \dfrac{\sin i_1}{\sin i_2}$ ---------------------------**(ii) (reciprocal of the above equation)**

Equating equation (i) and (ii) gives

$$_1n_2 = \frac{1}{_2n_1} \quad \textbf{OR} \quad _1n_2 \times {_2n_1} = 1$$

**CASE II:**

Consider a ray of light moving from **medium 1 (air)** through a series of media **2,3** and then finally emerge into **medium 1 (air)** as shown.




At **medium 2-medium 3** interface, Snell's law gives.

$$_2n_3 = \frac{\sin i_2}{\sin i_3} = \frac{\sin i_2}{\sin i_1} \times \frac{\sin i_1}{\sin i_3} = {_2n_1} \times {_1n_3}$$

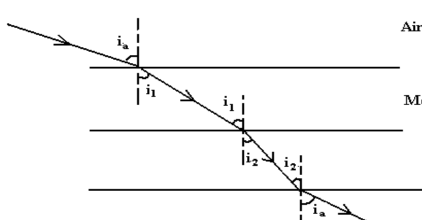
$$\therefore \quad _2n_3 = {_2n_1} \times {_1n_3}$$

Using the relation $_2n_1 = \dfrac{1.}{_1n_2}$

$$\Rightarrow \quad _1n_3 = {_1n_2} \times {_2n_3} \quad \textbf{OR} \quad _2n_3 = \frac{_1n_3}{_1n_2}$$

**GENERAL RELATION BETWEEN n AND sin i**

Consider a ray of light moving from air through a series of media **1, 2** and then finally emerge into air as shown.



At **air – medium 1** interface, Snell's gives $\frac{\sin i_a}{\sin i_1} = n_1$

$\Rightarrow$ $\mathbf{\sin i_a} = n_1 \sin i_1$ ---------------------------------------- ( i )

At **air - medium 2** interface, Snell's gives $\frac{\sin i_a}{\sin i_2} = n_2$

$\Rightarrow$ $\mathbf{\sin i_a} = n_2 \sin i_2$ -------------------------------------(ii)

Equating equation (i) and (ii) gives

$n_1 \sin i_1 = n_2 \sin i_2$.

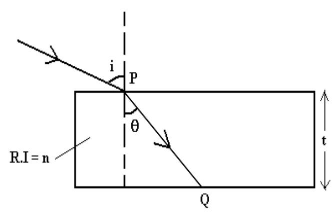
$\therefore$ **n sin i = a constant.**

**EXAMPLES:**

**1**. Monochromatic light incident on a block of material placed in a vacuum is refracted through an angle **θ**. If the block has a refractive index **n** and is of thickness **t**, show that this light takes a time $\frac{\mathbf{n\ t\ \sec\theta}}{\mathbf{c}}$ to emerge from the block.

where **c** is the speed of light in a vacuum.

Solution:




Let T be the time taken by light to travel from point P to Q in the medium.

Thus $T = \frac{\text{distance PQ}}{\text{speed of light in the block, } \mathbf{c_m}}$.

Where distance PQ $= \frac{t}{\cos\theta} = t \sec\theta$

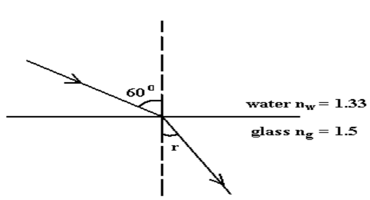
$\Rightarrow$ $T = \frac{t \sec\theta}{c_m}$ -------------------------------------(i)

By definition, $n = \frac{c}{c_m}$ $\Rightarrow$ $c_m = \frac{c}{n}$ ---------------(ii)

Substituting equation (ii) in to (i) gives

$T = \frac{\mathbf{n\ t\ \sec\theta}}{\mathbf{c}}$

**2**. A monochromatic beam of light is incident at **60°** on a water-glass interface of refractive index **1·33** and **1·5** respectively as shown




Calculate the angle of reflection **r.**

**Solution:**

Applying Snell's law gives

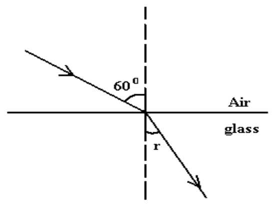
$n_w \sin 60° = n_g \sin r$

$1{\cdot}33 \sin 60° = 1{\cdot}5 \sin r$

**Thus $\angle r = 50{\cdot}2°$.**

**3.** A ray of light propagating from air is incident on an air-glass interface at an angle of **60°.** If the refractive index of glass is **1·5**, calculate the resulting angle of refraction.

Solution.



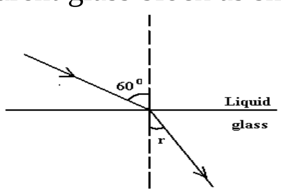

Applying Snell's law gives

$n_a \sin 60° = n_g \sin r$ but $n_a = 1$ , $n_g = 1{\cdot}5$

$\Rightarrow \quad 1 \sin 60° = 1{\cdot}5 \sin r$

$\therefore \quad \angle$ **r = 35·6°.**

**4**. A monochromatic ray of light is incident from a liquid on to the upper surface of a transparent glass block as shown.




Given that the speed of light in the liquid and glass is $\mathbf{2{\cdot}4 \times 10^8\ ms^{-1}}$ and $\mathbf{1{\cdot}92 \times 10^8\ ms^{-1}}$ respectively, find the angle of refraction, **r.**

Solution:

Applying Snell's law gives

$n_l \sin 60° = n_g \sin r$

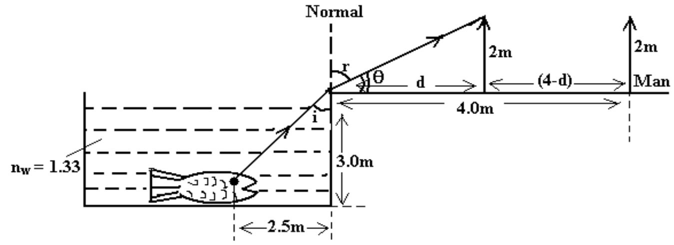
$\frac{c}{c_l} \sin 60° = \frac{c}{c_g} \sin r$

$\Rightarrow \quad \sin r = \frac{c_g}{c_l} \sin 60°$ but $c_g = 1{\cdot}92 \times 10^8$ ms $^{-1}$ , $c_l = 2{\cdot}4 \times 10^8$ ms $^{-1}$

$\therefore \quad \sin r = \frac{1{\cdot}92 \times 10^8}{2{\cdot}4 \times 10^8} \sin 60°$

$\Rightarrow \quad \angle$ **r = 43·9°.**

**5**. A small fish is 3·0m below the surface of the pond and 2·5m from the bank. A man 2·0m tall stands 4·0m from the pond. Assuming that the sides of the pond are vertical, calculate the distance the man should move towards the edge of the pond before movement becomes visible to the fish. **(Refractive index of water = 1·33).**




From the diagram, $\tan i = \frac{2\cdot5}{3}$ $\Rightarrow$ $\angle i = 39\cdot81°$

Applying Snell's law at the edge of the pond gives

$n_w \sin i = n_a \sin r$

$1\cdot33 \sin 39\cdot81° = 1 \sin r$

$\Rightarrow \angle r = 58\cdot4°$

Thus $\angle\theta = 90° - 58\cdot4° = 31\cdot6°$

From the diagram $\tan\theta = \frac{2}{d}$ $\Rightarrow$ $d = \frac{2}{\tan 31\cdot6°}$

$\therefore d = 3\cdot2m.$

Thus required distance traveled $= 4 - d$

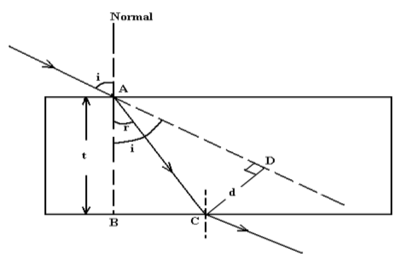
$= 4 - 3\cdot2$

$= 0\cdot8m$

**SIDE WISE DISPLACEMENT OF LIGHT RAYS.**

When light travels from one medium to another, its direction is displaced side ways. This is called lateral displacement.

Consider a ray of light incident at an angle **i** on the upper surface of a glass block of thickness **t**, and then suddenly refracted through an angle **r** causing it to suffer a sidewise displacement **d**.




From $\Delta$ ABC, $AC = \frac{t}{\cos r}$ --------------------------------.(i)

From the diagram, $\angle CAD = (i - r)$

From $\Delta$ ACD, $AC = \frac{d}{\sin(i - r)}$ ------------------------(ii)

Equating equation (i) and (ii) gives

$$\frac{t}{\cos r} = \frac{d}{\sin(i - r)}$$

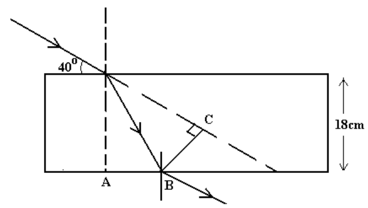
$$\Rightarrow \quad \mathbf{d = \frac{t \sin(i - r)}{\cos r}}$$

**NOTE :**

The horizontal displacement of the incident ray , **BC = t. tan r**

**Example:**

The figure below shows a monochromatic ray of light incident from a liquid of refractive index 1·33 onto the upper surface of a glass block of refractive index 1·42

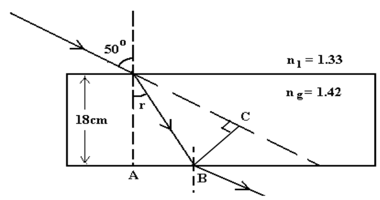



Calculate the;

(i) horizontal displacement AB.

(ii) lateral displacement BC of the emergent light.

**Solution**




**(i)** Applying Snell's law at the liquid- glass interface gives,

$$n_l \sin 50° = n_g \sin r.$$

$$1{\cdot}33 \sin 50° = 1{\cdot}42 \sin r$$

$$\Rightarrow \quad \angle r = 45{\cdot}8°$$

**Horizontal displacement AB =** t tan r

= 18 tan 45·8°

**= 18·51cm**

**(ii) Lateral displacement d** $= \dfrac{t \sin(i - r)}{\cos r}$

$$d = \frac{18 \sin(50° - 45{\cdot}8°)}{\cos 45{\cdot}8°}.$$

$$\mathbf{d = 1{\cdot}89cm.}$$

**EXERCISE**

**(1)** What is meant by **refraction of light?**

**(2) (i)** State the laws of refraction of light.

**(ii)** State what brings about refraction of light as it travels from one medium to another.

**(3) (i)** What is meant by the **refractive index** of a material**?**

**(ii)** Light of two colours blue and red is incident at an angle $\gamma$ from air to a glass block of thickness $\mathbf{t}$. When blue and red lights are refracted through angles of $\theta_b$ and $\theta_r$ respectively, their corresponding speeds in the glass block are $v_b$ and $v_r$. Show that the separation of the two colours at the bottom of the glass block $\mathbf{d} = \frac{t}{c}\left[\frac{v_r}{\cos\theta_r} - \frac{v_b}{\cos\theta_b}\right]\sin\gamma$ Where $\theta_r > \theta_b$ and $\mathbf{c}$ is the speed of light in air.

**(iii)** Light consisting of blue and red is incident at an angle of $\mathbf{60°}$ from air to a glass block of thickness $\mathbf{18cm.}$ If the speeds of blue and red light in the glass block are $1\cdot86\times10^8 ms^{-1}$ and $1\cdot92\times10^8 ms^{-1}$ respectively, find the separation of the two colours at the bottom of the glass block..

**[Answer: 0·54cm]**

**(4)** Show that when the ray of light passes through different media separated by plane boundaries, **n sinϕ = constant** where **n** is the absolute refractive index of a medium and $\phi$ is the angle made by the ray with the normal in the medium.

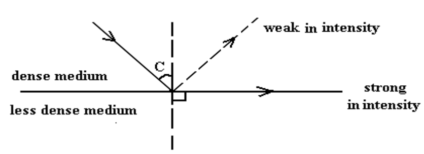
**(5)** Show that when the ray of light passes through different media **1** and **2** separated by plane boundaries, $_1n_2 \times {}_2n_1 = 1$ where **n** is the refractive index of a medium.

**(6).** Show that when the ray of light passes through different media **1,2** and **3** separated by plane boundaries, $_1n_3 = {}_1n_2 \times {}_2n_3$ where **n** is the refractive index of a medium.

**(7)** Show that a ray of light passing through a glass block with parallel sides of thickness **t** suffers a sidewise displacement $\mathbf{d} = \frac{t\sin(\phi-\lambda)}{\cos\lambda}$, where $\phi$ is the angle of incidence and $\lambda$ is the angle of refraction.

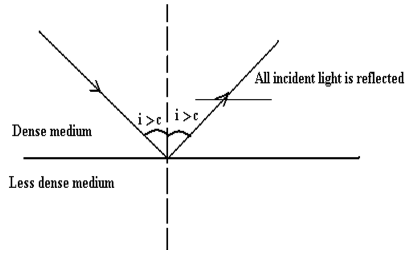
**CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION**

Consider a ray of light incident on an interface between two media as shown.




**critical angle**: is the angle of incidence in the dense medium which results into an angle of refraction of **90°** in the adjoining less dense medium.

If the angle of incidence is greater than the critical angle, all the incident light energy is reflected back in the dense medium and **total internal refection** is said to have occurred.




**NOTE :**

**(a)** At critical point Snell's law becomes.

$n_1 \sin c = n_2 \sin 90°$

$\Rightarrow \quad \sin c = \frac{n_2}{n_1}$

$$\therefore \quad \mathbf{c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

Where $\mathbf{n_1}$ and $\mathbf{n_2}$ are the refractive indices of the dense and the less dense medium respectively.

However if the less dens medium is air, then Snell's law becomes

$n_1 \sin c = n_a \sin 90°$

$\Rightarrow \quad \sin c = \frac{1}{n_1}.$

**(b)** The conditions for total internal reflection to occur are;

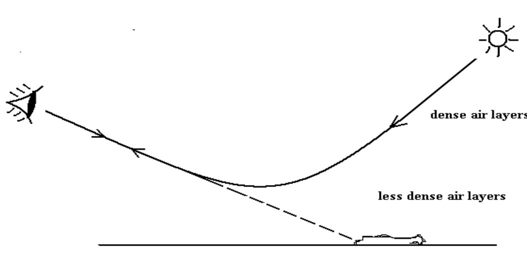
(i) Light should travel from an optically denser to a less dense medium.

(ii) The angle of incidence at the boundary of the media should be greater than the critical angle.

**APPLICATION OF TOTAL INTERNAL REFLECTION.**

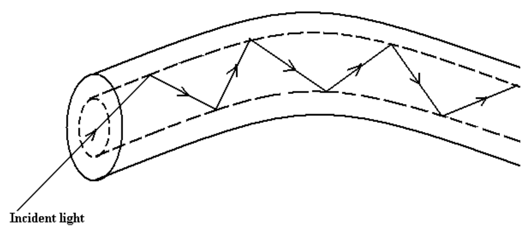
**(i)** It is responsible for the formation of a mirage.
**(ii)** It is responsible for the formation of a rainbow.
**(iii)** It is responsible for the transmission of light in optical fibres.
**(iv).** It is responsible for the transmission of sky radio waves
**(v).** It is responsible for the transmission of light in prism binoculars.

**FORMATION OF A MIRAGE**



On a hot day, The air layers near the earth's surface are hot and are less denser than the air layers above the earth's surface. Therefore as light from the sky pass through the various layers of air, light rays are continually refracted away from the normal till some point where light is totally internally reflected. An observer on earth receiving the totally internally reflected light gets an impression of a pool of water on the ground and this is the virtual image of the sky.

**AN OPTICAL FIBRE**



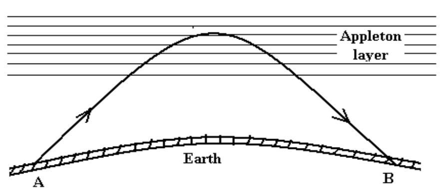

When a light ray enters in to the fibre, it bounces from one edge to another by total internal reflection. These successive total internal reflections enable the transmission of light in optical fibres.

**NOTE**

**(i)** An optical fibre finds a practical application in an endoscope, a device used by doctors to inside the human body.

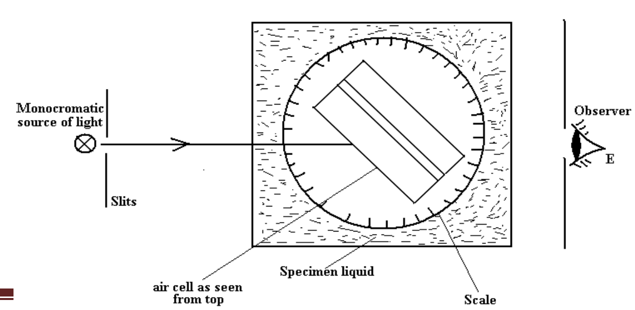
**(ii)** Optical fibres are used in telecommunication systems (i.e. Telephone or TV signals are carried along optical fibers by laser light).

**SKY RADIO WAVES**




A radio wave sent skyward from a station transmitter **A** is continually refracted away from the normal on entering the electron layer that exists above the earth's surface. Within the electron layer, the wave is totally internally reflected causing it to emerge from the electron layers and finally returns to the earth's surface where it's presence can be detected by a radio receiver at B

**DETERMINATION OF REFRACTIVE INDEX OF A LIQUID BY AN AIR-CELL METHOD**

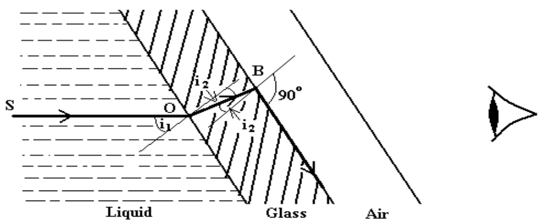


The air cell is immersed in a liquid under test. A beam of monochromatic light is directed onto the air cell and then observed through the cell from the opposite side at **E**. The cell is then rotated on one side until light is suddenly cut off and the angular position $\boldsymbol{\theta_1}$ is noted. The cell is again rotated in the opposite direction until light is suddenly cut off and the angular position $\boldsymbol{\theta_2}$ is noted. The refractive index of the liquid can then be calculated from $\mathbf{n} = \dfrac{1}{\sin \boldsymbol{\theta}}$ where $\boldsymbol{\theta} = \dfrac{\boldsymbol{\theta_1} + \boldsymbol{\theta_2}}{\mathbf{2}}$

NB:

The source of light should be monochromatic so that the extinction of light is sharp since monochromatic light does not under go dispersion, as it is with white light.

**THEORY OF THE AIR-CELL METHOD**




Ray **OS** is refracted along **OB** in glass. However, at **B** total internal just begins.
Suppose $\mathbf{i_1}$ is the angle of incidence in the liquid, $\mathbf{i_2}$ is the angle of incidence in the glass while **n** and $\mathbf{n_g}$ are the corresponding refractive indices, Then applying the relation **n sin i = a constant** gives

$n \sin i_1 = n_g \sin i_2 = n_a \sin 90°$

$\therefore \quad n \sin i_1 = n_a \sin 90°$

$\Rightarrow n \sin i_1 = 1$

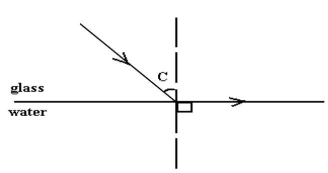
**Thus** $\mathbf{n = \dfrac{1}{\sin i_1}}$

But $\angle i_1 = \dfrac{\theta_1 + \theta_2}{2}$ Hence $n = \dfrac{1}{\sin \theta}$

**Examples:**

**1** The critical angle for water-air interface is $48° \; 42^1$ and that of glass-air interface is $38° \; 47^1$. Calculate the critical angle for glass-air interface.

Solution




Applying Snell's law g ------------------(i).

Given that for water-air interface $c_w = 48°42^1$.

$\therefore \; n_w \sin c_w = 1$

$n_w \sin 48°42^1 = 1$

$\Rightarrow \quad n_w = 1{\cdot}33$------------------------------------------------------(ii)

Also for glass-air interface $c_g = 38°47^1$

$n_g \sin c_g = 1$

$\therefore \quad n_g \sin 38°47^1 = 1$

$\Rightarrow \quad n_g = 1{\cdot}67$----------------------------------------------(iii)

Substituting equation (ii) and (iii)into equation (i) gives

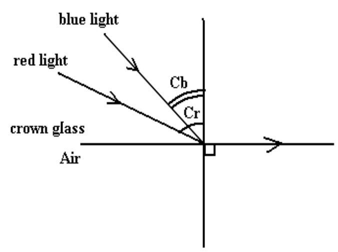
$1{\cdot}67 \sin c = 1{\cdot}33 \sin 90°$

$\Rightarrow \quad c = 52{\cdot}8°$

**2.** The refractive index for red light is 1·634 of crown glass and the difference between the critical angles of red and blue light at the glass-air interface is $0°56^1$.What is the refractive index of crown glass for blue light

**Solution**

Analysis the critical angle between two media for red light is greater than that for any other light colour. This gives rise to the ray diagram below




Since $c_r > c_b$ , then $c_r - c_b = 0°56^1$ ----------------------------(i)

Applying Snell's law to red light gives

$n_r \sin c_r = 1$

$1{\cdot}63 \sin c_r = 1$

$\Rightarrow \quad c_r = 37{\cdot}73°$

Equation (i) now becomes

$c_b = c_r - 0°56^1$

$c_b = 37{\cdot}73° - 0°56^1$

$c_b = 36{\cdot}8°$

Applying Snell's law to blue light gives

$n_b \sin c_b = 1$

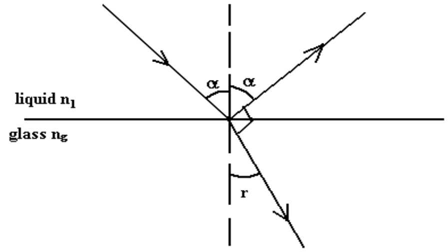
$n_b \sin 36{\cdot}8° = 1$

$\Rightarrow \quad n_b = 1{\cdot}67$

**2. (i)** A glass block of refractive index $n_g$ is immersed in a liquid of refractive index $n_l$. A ray of light is partially reflected and refracted at the interface such that the angle between the reflected and the refracted ray is 90°. Show that $\mathbf{n_g = n_l \tan \alpha}$ where $\alpha$ is the angle of incidence from the liquid to glass.

**(ii)** When the procedures in **(i)** above are repeated with the liquid removed, the angle of incidence increases by **8°**. Given that $\mathbf{n_l = 1{\cdot}33}$**,** find $\mathbf{n_g}$ and the angle of incidence at the liquid-glass interface.

Solution




Applying Snell's law at the liquid-glass interface gives

$$n_g \sin r = n_l \sin \alpha$$

But $r + 90° + \alpha = 180°$

$\Rightarrow r = 90° - \alpha$

$\therefore n_g \sin(90° - \alpha) = n_l \sin \alpha.$

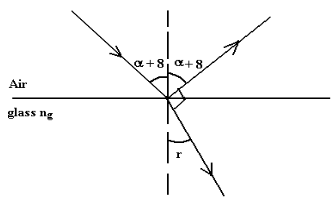
From trigonometry, $\sin(90° - \alpha) = \cos \alpha$

$\Rightarrow n_g \cos \alpha = n_l \sin \alpha.$

Dividing $\cos \alpha$ on both sides gives

$$\mathbf{n_g = n_l \tan \alpha.}$$

**(ii) When the liquid is removed.**




From $n_g = n_l \tan \alpha$------------------------------------------------------(i)

$\Rightarrow n_g = n_a \tan(\alpha + 8°)$ but $n_a = 1$

$$\therefore n_g = \frac{\tan \alpha + \tan 8°}{1 - \tan \alpha \tan 8°}$$

$n_g(1 - \tan \alpha \tan 8°) = \tan \alpha + \tan 8°$

$n_g - n_g \tan \alpha \tan 8° = \tan \alpha + \tan 8°$----------------------------------(ii)

from equation (i) $\tan \alpha = \dfrac{n_g}{n_l}$

Substituting for $\tan \alpha$ in equation (ii) gives

$$n_g - \frac{n_g^2 \tan 8°}{n_l} = \frac{n_g}{n_l} + \tan 8°$$

but $n_l = 1{\cdot}33.$

$\therefore n_g^2 - 2.340\, n_g + 1{\cdot}326 = 0$ -----------------------------(iii)

Equation (iii) is quadratic in $n_g$ and solving it gives

$n_g = 1{\cdot}39$ **or** $n_g$ not physically possible.

Using equation (i)

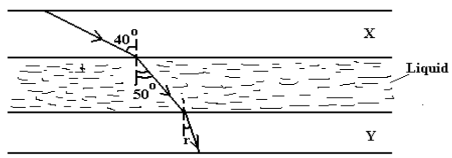
$$\tan \alpha = \frac{ng}{n_l} = \frac{1{\cdot}39}{1{\cdot}33}$$

$$\alpha = \tan^{-1}(1.045) = 46{\cdot}3°$$

**The required angle of incidence** = $\alpha + 8°$

$= \mathbf{54{\cdot}3°}$

**3**. The figure below shows a liquid layer confined between two transparent plates **X** and **Y** of refractive index **1·54** and **1·44** respectively.




A ray of monochromatic light making an angle of **40°** with the normal to the interface between media **X** and the liquid is refracted through an angle of **50°** by the liquid. Find the

(i) refractive index of the liquid.

(ii) angle of refraction ,**r** in the medium **Y.**

(iii) minimum angle of incidence in the medium **X** for which the light will not emerge from medium **Y**.

**Solution**

**(i)** Applying Snell's law at the plate **X** – liquid interface gives

$$n_x \sin 40° = n_l \sin 50°$$

$$1{\cdot}54 \sin 40° = n_l \sin 50°$$

$$n_l = \frac{1{\cdot}54 \sin 40°}{\sin 50°}$$

$$\therefore \ \mathbf{n_l = 1{\cdot}29}$$

**(ii)** Applying Snell's law at the liquid – plate **Y** interface gives

$$n_l \sin 50° = n_y \sin r$$

$$1{\cdot}29 \sin 50° = 1{\cdot}44 \sin r$$

$$\Rightarrow \ \angle \mathbf{r = 43{\cdot}3°}$$

**(iii)** For light not to emerge from plate Y, it grazes the liquid – plate Y interface.

$$\Rightarrow \ \angle r = 90°$$

Applying Snell's law at the liquid – plate Y interface gives

$$n_l \sin i_l = n_y \sin 90°$$

$$1{\cdot}29 \sin i_l = 1{\cdot}44 \sin 90°$$

$$\sin i_l = \frac{1{\cdot}44}{1{\cdot}29} \text{ --------------------------------(i)}$$

More over, applying Snell's law at the plate X – liquid interface gives

$$n_x \sin i_x = n_l \sin i_l$$

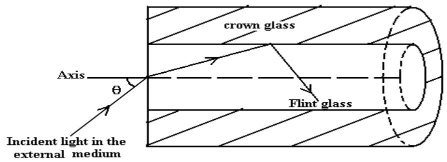
$$1{\cdot}54 \sin i_x = 1{\cdot}29 \sin i_l \text{ ----------------------------(ii)}$$

Substituting equation (i) in (ii) gives

$$1{\cdot}54 \sin i_x = 1{\cdot}29 \times \frac{1{\cdot}44}{1{\cdot}29}$$

$$\Rightarrow \ \angle \mathbf{i_x = 40{\cdot}5°}$$

**5.**The diagram below shows a cross-section through the diameter of the light pipe with an incident ray of light in its plane.

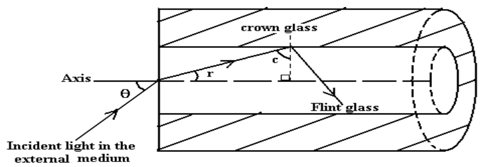



The refractive indices for flint glass, crown glass and the external medium are $n_1$, $n_2$ and $n_3$ respectively. Show that a ray that enters the pipe is totally reflectedat the flint - crown glass interface provided $\mathbf{sin\theta} = \dfrac{\sqrt{\mathbf{n_1^2 - n_2^2}}}{\mathbf{n_3}}$ where

$\theta$ is the maximum angle of incidence in the external medium.

Solution

Analysis for light to be totally reflected, it must be incident at a critical angle on the flint-crown glass interface




Applying Snell's law at the external medium-flint glass interface gives

$$n_3 \sin\theta = n_1 \sin r$$

but $r + c = 90°$

$\therefore \quad r = 90° - c$

$\Rightarrow \quad n_3 \sin\theta = n_1 \sin(90° - c)$

$\therefore \quad n_3 \sin\theta = n_1 \cos c$

$\Rightarrow \quad \cos c = \dfrac{n_3 \sin\theta.}{n_1}$ --------------------------------(i)

Applying Snell's law at the flint-crown glass interface gives

$$n_1 \sin c = n_2 \sin 90°$$

$\Rightarrow \quad \sin c = \dfrac{n_2}{n_1}$ ------------------------------------------------------------(ii)

Using the trigonometrical relation $\sin^2 c + \cos^2 c = 1$, then

$$\left(\frac{n_2}{n_1}\right)^2 + \left(\frac{n_3 \sin\theta}{n_1}\right)^2 = 1$$

**Thus,** $\mathbf{sin\theta} = \dfrac{\sqrt{\mathbf{n_1^2 - n_2^2}}}{\mathbf{n_3}}$

**EXERCISE**

**1.** Explain the term total internal reflection and give three instances where it is applied.

**2**. With the aid of suitable ray diagrams, explain the terms critical angle and total internal reflection.

**3.** Show that the relation between the refractive index **n** of a medium and critical angle **c** for a ray of light traveling from the medium to air is given by **n** = 1

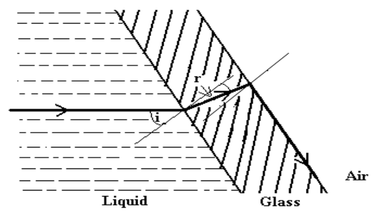
**4.**Show that the critical angle, **c** at a boundary between two media when light travels from medium 1 to medium 2 is given by $\sin c = \frac{n_2}{n_1}$ where $n_1$ and $n_2$ are the refractive indices of the media respectively.

**5.** Explain how a mirage is formed.

**6.**Explain briefly how sky radio waves travel from a transmitting station to a receiver.

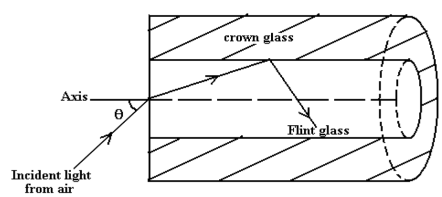
**7.** Describe how you would determine the refractive index of the liquid using an air cell.

**8)**



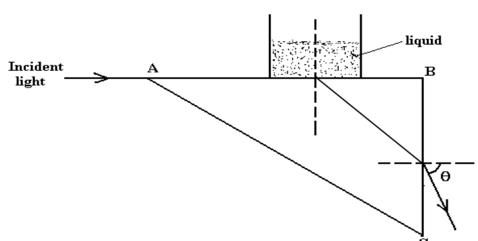

In the figure above, a parallel sided glass slide is in contact with a liquid on one side and air on the other side. A ray of light incident on glass slide from the liquid emerges in air along the glass-air interface. Derive an expression for the absolute refractive index **,n,** of the liquid in terms of the angle of incidence **i** in the liquid-medium.

**(9)** The diagram below shows a cross-section through the diameter of the light pipe with an incident ray of light in its plane.




The refractive indices for flint glass and crown glass are $n_1$ and $n_2$ respectively. Show that a ray which enters the pipe is totally reflected at the flint-crown glass interface provided $\sin\theta = \sqrt{n_1^2 - n_2^2}$ where $\theta$ is the maximum angle of incidence at the air-flint glass interface

**10**.A liquid of refractive index $\mu_L$ is tapped in contact with the base of a right-angled prism of refractive index $\mu_g$ by means of a transparent cylindrical pipe as shown.

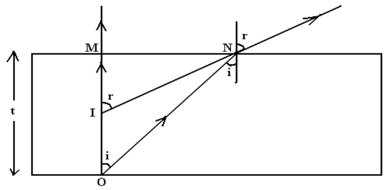



Show that a ray of light which is at a grazing incidence on the liquid-glass interface emerges in to air through face **BC** at an angle θ below the horizontal provided

$\mu_L = \sqrt{\mu_g^2 - \sin^2\theta}$ . Hence find $\mu_L$, if $\mu_g = 1{\cdot}52$ and $\theta = 47{\cdot}4°$

**REAL AND APPARENT DEPTH**

Consider an object **O** viewed normally from above through a parallel-sided glass block of refractive index $\mathbf{n_g}$ and thickness **t** as shown.




A ray from an object **O** normal to the glass surface at **M** passes un deviated. While the ray **ON** inclined at a small angle **i** to the normal is refracted at **N**. The observer above the glass block sees the image of the object **O** at **I.**

Applying Snell's law at **C** gives

$n_g \sin i = n_a \sin r$------------------------------------(i). From the diagram, $\sin i = \frac{MN}{ON}$ and $\sin r = \frac{MN}{IN}$

equation (i) now becomes $n_g \frac{MN}{ON} = \frac{MN}{IN}$

$$\Rightarrow \quad n_g = \frac{ON}{IN}$$

Since angle **i** is very small, then $ON \approx OM$ and $IN \approx IM$

$$\Rightarrow \quad n_g = \frac{OM}{IM}$$

From the diagram**, OM = real depth** and **IM = apparent depth**.

Hence $\mathbf{n_g = \dfrac{real\ depth}{apparent\ depth}}$.

If the apparent displacement of the object **OI = d,** then **IM = ( t – d ).**

So that $n_g = \frac{OM}{IM} = \frac{t}{t-d}$

**Thus apparent displacement** $\mathbf{d = \left(1 - \dfrac{1}{n_g}\right)t}$

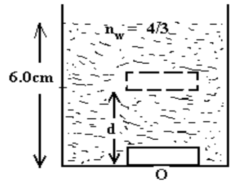
**NOTE**

**(i)** The apparent displacement **d** of an object **O** is independent of the position of **O** below the glass block. Thus the same expression above gives the displacement of an object which is some distance in air below a parallel-sided glass block.

**(ii)** If there are different layers of different transparent materials resting on top of each other, the apparent position of the object at the bottom can be found by adding the separate displacements due to each layer.

**EXAMPLES:**

**1**.An object at a depth of **6·0cm** below the surface of water of refractive index $\mathbf{4/3}$ is observed directly from above the water surface. Calculate the apparent displacement of the object



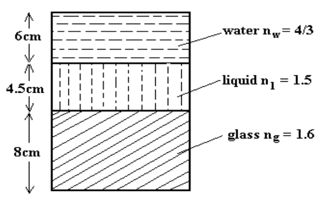

Using the relation $\mathbf{d} = \left(\mathbf{1 - \frac{1}{n_g}}\right)\mathbf{t}$ gives

$$d = \left(1 - \frac{3}{4}\right) \times 6$$

$$\mathbf{d = 1{\cdot}5\ cm}$$

**2.** A tank contains a slab of glass **8cm** and refractive index **1·6**. Above this is a depth of **4·5cm** of a liquid of refractive index **1·5** and upon this floats **6cm** of water of refractive index $\mathbf{4/3}$ calculate the apparent displacement of an object at the bottom of the tank to an observer looking down wards directly from above.

**Solution**




Apparent displacement $\mathbf{d = d_w + d_l + d_g}$

Using the relation $d = \left(1 - \frac{1}{n}\right) t$

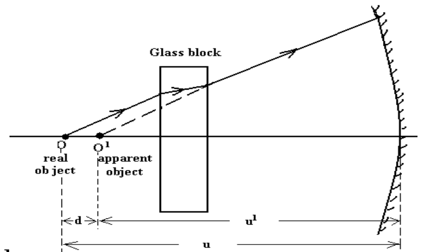
$$\therefore\ \mathbf{d} = \left(\mathbf{1 - \frac{1}{n_w}}\right)\mathbf{t_w} + \left(\mathbf{1 - \frac{1}{n_l}}\right)\mathbf{t_l} + \left(\mathbf{1 - \frac{1}{n_g}}\right)\mathbf{t_g}$$

$$\Rightarrow \quad \mathbf{d} = \left(1 - \frac{3}{4}\right) \times 6 + \left(1 - \frac{1}{1\cdot5}\right) \times 4\cdot5 + \left(1 - \frac{1}{1\cdot6}\right) \times 8$$

$$\mathbf{d = 1{\cdot}5 + 1{\cdot}5 + 3}$$

$$\mathbf{d = 6cm.}$$

**3.**A small object is placed **20cm** in front of a concave mirror of focal length **15cm.** A parallel-sided glass block of thickness **6cm** and refractive index **1·5** is then placed between the mirror and the object. Find the shift in the position and size of the image




Consider the action of a concave mirror in the absence of a glass block

u = 20cm and f = 15cm

using the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ gives

$$v = \frac{fu}{u - f} = \frac{15 \times 20}{20 - 15} = 60\text{cm}$$

Thus in the absence of a glass block**, image distance = 60cm**

In this case**, magnification** $\mathbf{m} = \frac{\mathbf{v}}{\mathbf{u}} = \frac{\mathbf{60}}{\mathbf{20}} = \mathbf{3}$

Consider the action of a glass block

Apparent displacement $\mathbf{d} = \left(\mathbf{1 - \frac{1}{n_g}}\right)\mathbf{t} = \left(\mathbf{1 - \frac{1}{1\cdot5}}\right)\times\mathbf{6} = \mathbf{2cm}$

Thus in the presence of a glass block, object distance $\mathbf{u^1 = (20 - 2)}$ **cm =18cm**

**"The object is displaced and it appears to be 18cm in front of the mirror"**

Consider the action of a concave mirror in the presence of a glass block

$u^1$ = 18cm and f = 15cm

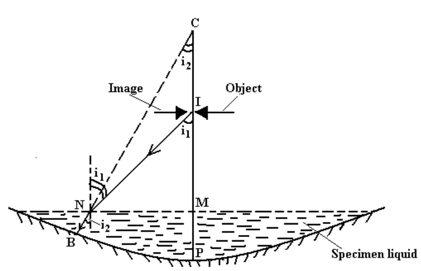
using the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ gives

$$v^1 = \frac{fu^1}{u^1 - f} = \frac{15 \times 18}{18 - 15} = 90\text{cm}$$

$\therefore$ **The required shift in the image position =** $\mathbf{v^1 - v}$ **= (90 – 60) cm = 30cm**

The magnification now becomes $\mathbf{m^1} = \frac{\mathbf{v^1}}{\mathbf{u^1}} = \frac{\mathbf{90}}{\mathbf{18}} = \mathbf{5}$

**DETERMINATION OF REFRACTIVE INDEX OF A LIQUID USING A CONCAVE MIRROR METHOD.**




A small quantity of the liquid under test is poured into a concave mirror of known radius of curvature **r**. An object pin is moved along the principal axis of the mirror until it coincides with its image at **I.** The distance **IP** is noted. The required refractive index $\mathbf{n_l} = \underline{\mathbf{r}}$

**IP.**

**PROOF**

For refraction at **N**, $n_l \sin i_2 = n_a \sin i_1$ --------------------------- (i)

From the diagram, $\sin i_2 = \frac{NM}{NC}$ and $\sin i_1 = \frac{MN}{NI}$

Equation (i) now becomes

$$\mathbf{n_1} \frac{NM}{NC} = \frac{NM}{NI}.$$

On simplifying, $\mathbf{n_l} = \frac{NC}{NI}$

But **N** is very close to **M** hence NC ≈ MC and NI ≈ MN

$$\Rightarrow \qquad \mathbf{n_l} = \frac{MC}{MI}$$

Also for a small quantity of the liquid, **M** is close to **P** ⇒ MC ≈ CP = **r**, and MI ≈ IP.

$$\textbf{Thus} \quad \mathbf{n_l} = \frac{\mathbf{r}}{\mathbf{IP}}.$$

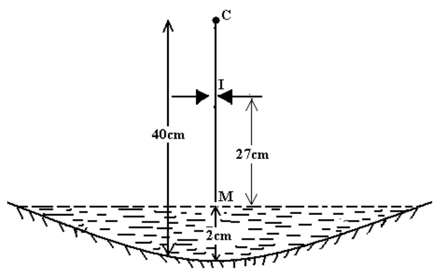
**NOTE :**

**(i)** If the specimen liquid is of reasonable quantity, then its depth **d** can not be ignored. In this case, $\mathbf{n_l} = \frac{\mathbf{MC}}{\mathbf{MI}} = \frac{\mathbf{r - d}}{\mathbf{MI}}$

**(ii)** if the radius of curvature **r** of the concave mirror is not known, first determine it using the method discussed in the previous section.

**EXAMPLES:**

1.A liquid is poured in to a concave mirror to a depth of **2·0cm**. An object held above the liquid coincides with its own image when its **27·0cm** above the liquid surface. If the radius of curvature of the mirror is **40·0cm**, calculate the refractive index of the liquid.

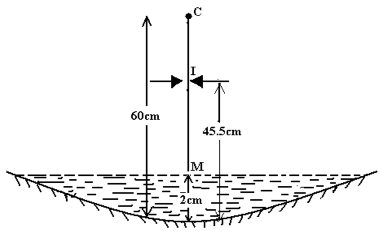



Using the relation $\mathbf{n_l} = \frac{\mathbf{r - d}}{\mathbf{MI}}$

$$= \frac{40 - 2}{27}$$

$$\therefore \qquad \mathbf{n_l = 1{\cdot}4}$$

**2**. A liquid is poured into a concave mirror to a depth of **2·0cm**. An object held above the liquid coincides with its image when it is **45·5cm** from the pole of the mirror. If the radius of curvature of the mirror is **60·0cm**, calculate the refractive index of the liquid.




Using the relation $n_l = \dfrac{\mathbf{r - d}}{\mathbf{MI}}$

$$= \frac{60 - 2}{45{\cdot}5 - 2}$$

$$\therefore \quad \mathbf{n_l = 1{\cdot}33}$$

**3**. A small concave mirror of focal length **8cm** lies on a bench and a pin is moved vertically above it .At what point will the pin coincide with its image if the mirror is filled with water of refractive index **4/3.**

**Solution**

**ANALYSIS**

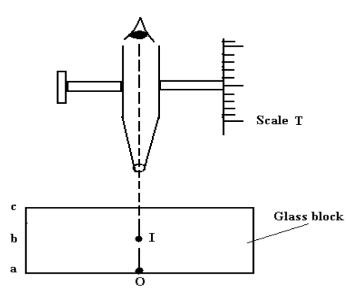
For a small concave mirror, the quantity of water is small that its depth **d** can be ignored

Using the relation $n_l = \dfrac{\mathbf{r}}{\mathbf{IP}}$ Where **r = 2f = 2 × 8 = 16cm**

$$\Rightarrow \quad \mathbf{IP} = \frac{r}{n_1} = \frac{16}{4/3} = \mathbf{12cm}$$

Therefore the pin coincided with its image at a height of **12cm** above the mirror

**MEASUREMENT OF REFRACTIVE INDEX OF A GLASS BLOCK BY APPARENT DEPTH METHOD.**



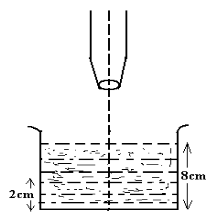

A vertically traveling microscope having a graduated scale **T** besides it is focused on lycopodium particles placed at **O** on a sheet of white paper. The scale reading **a** on **T** is noted. A glass block whose refractive index is required is placed on a paper and the microscope is raised until the particles are refocused at **I**. The scale reading **b** is again noted. Lycopodium particles are then poured on top of the glass block and the microscope is re-raised until the particles are again refocused. The new scale reading **c** is then noted. The refractive index of the block can then be calculated from

$$n = \frac{\textbf{Real depth}}{\textbf{Apparent depth}} = \frac{|c - a|}{|c - b|}$$

**NOTE**

The refractive index of a liquid can be found by focusing on a sand particle at the bottom of an empty container and the scale reading is noted. The specimen liquid is then put in the container and the traveling microscope is refocused on the sand giving a scale reading b. Finally the traveling microscope is focused on the liquid surface giving a scale reading c. Thus $\mathbf{n_L} = \frac{|c-a|}{|c-b|}$

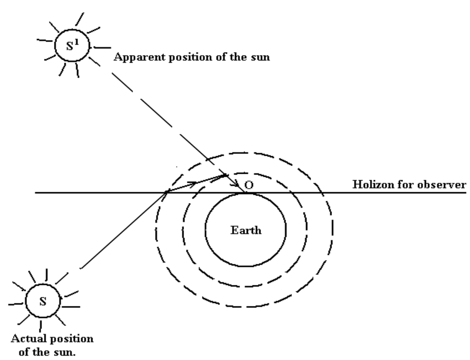
**EXAMPLE:** A microscope is focused on a mark at the bottom of the beaker. Water is poured in to the beaker to a depth of **8cm** and it is found necessary to raise the microscope through a vertical distance of **2cm** to bring the mark again in to focus. Find the refractive index of water.




**Using the relation** $\mathbf{n_g} = \frac{\text{real depth}}{\text{apparent depth}} = \frac{8}{8-2}$

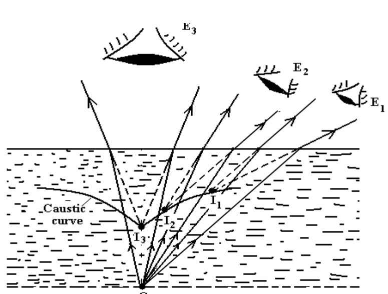
$\Rightarrow$ $\mathbf{n_g = 1{\cdot}33}$ **(to 2 decimal places )**

**OBSERVATION OF SUNLIGHT SITUATED BELOW THE HORIZON.**



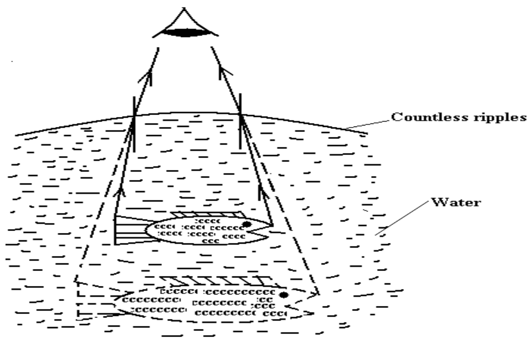

Light from the sun **S** is successively refracted from the different layers of the atmosphere due to the different optical densities they have. To an observer **O** on the earth's surface, sunlight appears to have come from point **S¹**.

**THE APPARENT SHAPE OF THE BOTTOM OF A POOL OF WATER**



When the observer moves the eye from $E_1$ towards $E_3$ the apparent position of **O** moves from $I_1$ to $I_3$ along a curve due to refracted rays bending away from the normal as shown above. Thus the bottom of the pool of water appears curved, being shallower near the edge than at the centre.

**THE APARENT SIZE OF A FISH SITUATED IN WATER.**




A large surface of water is not completely flat but consists of count less ripples whose convex surface on air acts as a convex lens of long focal length. In consequence the fish is with in the focal length of the lens hence it appears magnified to an observer viewing it from above.

**EXERCISE**

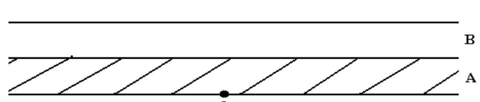
**1**. Show that for an object viewed normally from above through a parallel sided glass block , the refractive index of the glass material is given by

$$n_g = \frac{\textbf{real depth}}{\textbf{apparent depth}}$$

**2.** Derive an expression for the apparent displacement of an object when viewed normally through a parallel sided glass block.

**3**. A vessel of depth **2d cm** is half filled with a liquid of refractive index $\mu_1$,and the upper half is occupied by a liquid of refractive index $\mu_2$. Show that the apparent depth of the vessel, viewed perpendicularly is $\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)d$

**4**.Two parallel sided blocks **A** and **B** of thickness **4·0cm** and **5·0cm** respectively are arranged such that **A** lies on an object **O** as shown in the figure below



Calculate the apparent displacement of **O** when observed directly from above, if the refractive indices of **A** and **B** are **1·52** and **1·66**.

**5.** A tank contains liquid **A** of refractive index **1·4** to a depth of **7·0cm**. Upon this floats **9·0cm** of liquid **B**. If an object at the bottom of the tank appears to be **11·0cm** below the top of liquid **B** when viewed directly above from, calculate the refractive index of liquid **B**.

**6**. Describe how the refractive index of a small quantity of a liquid can be determined using a concave mirror.

**7.** Describe how the refractive index of a glass block can be determined using the apparent depth method.

**8.** A small liquid quantity is poured into a concave mirror such that an object held above the liquid coincides with its image when it is at a height **h** from the pole of the mirror. If the radius of curvature of the mirror is **r**, show with the aid of a suitable illustration, that the refractive index of the liquid. $\mathbf{n} = \frac{\mathbf{r}}{\mathbf{h}}$

**9.**Explain how light from the sun reaches the observer in the morning before the sun appears above the horizon

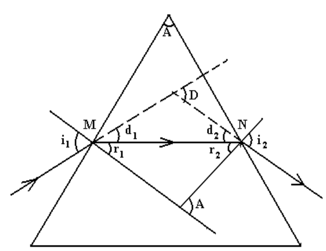
**10.** Explain the apparent shape of the bottom of a pool of water to an observer at the bank of the pool.

**11.** Explain why a fish appears bigger in water than its actual size when out of water.

**DEVIATION OF LIGHT THRUOGH A PRISM**

The angle of deviation caused by the prism is the angle between the incident ray and the emergent ray.

Consider a ray of light incident in air on a prism of refracting angle **A** and finally emerges into air as shown.




From the diagram above, MS and NS are normals at the points of incidence and emergence of the ray respectively.

$\therefore \quad \angle MPN + \angle MSN = 180°$

Thus $\angle NST = \angle MPN = A$

Suppose $\mathbf{i_1, r_1}$ and $\mathbf{i_2, r_2}$ represents angles of incidence and refraction at faces M and N respectively, then

From geometry of $\Delta$ MNS, $r_1 + r_2 = A$ ------------------ (i )

Total deviation $D = d_1 + d_2$ where $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

$\Rightarrow \quad D = i_1 - r_1 + i_2 - r_2$

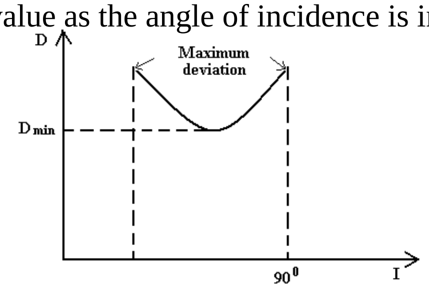
On simplifying, $D = i_1 + i_2 - (r_1+r_2)$ ------------- (ii)

Combining equation (i) and (ii) gives

$$D = i_1 + i_2 - A$$

**NOTE:**

Experiments show that as the angle of incidence **i** is increased from zero, the deviation **D** reduces continuously up to a minimum value of deviation $D_{min}$ and then increases to a maximum value as the angle of incidence is increased as shown below:

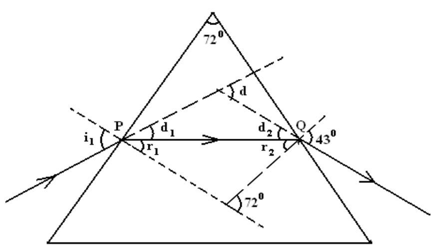



**EXAMPLE:**

**1**.A ray of light is incident on a prism of refracting angle **72°** and refractive index of **1·3.** The ray emerges from the prism at **43°.** Find

(i) the angle of incidence.

(ii) the deviation of the ray.




(i) At **P,** Snell's law becomes .

$$n_a \sin 43° = 1·3 \sin r_2$$
$$\therefore \quad r_2 = 31·64°$$
$$\text{But } r_1 + r_2 = 72°$$
$$\Rightarrow \quad r_1 = 72° - r_2$$
$$= 72° - 31·64°$$
$$\therefore \quad r_1 = 40·36°$$

At **Q**, Snell's law becomes

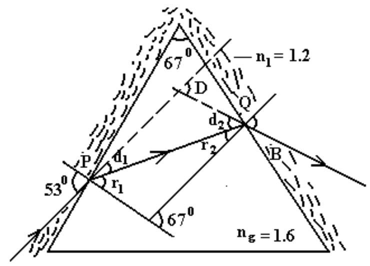
$$n_a \sin i_1 = 1·3 \sin 40·36°$$
$$\therefore \quad \mathbf{i_1 = 57·34°.}$$

(ii) **Total Deviation** $\mathbf{D = d_2 + d_1}$

where $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

$$\Rightarrow \quad D = (43° - 31·64°) + (57·34° - 40·36°)$$
$$\therefore \quad \mathbf{D = 28·34°.}$$

**2.**A prism of refracting angle 67° and refractive index of 1·6 is immersed in a liquid of refractive index 1·2. If a ray of light traveling through the liquid makes an angle of incidence of 53° at the left face of the prism, Determine the total deviation of the ray.




**Total Deviation $D = d_2 + d_1$**

where $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

$\Rightarrow$ $D = (53° - r_1) + (i_2 - r_2)$ ----------------------------------(i)

At P, Snell's becomes

$1{\cdot}2 \sin 53° = 1{\cdot}6 \sin r_1$

$\therefore$ $r_1 = 36{\cdot}8°$

But $r_1 + r_2 = 67°$

$\Rightarrow$ $r_2 = 67° - r_1$

$= 67° - 36{\cdot}8°$

$\therefore$ $r_2 = 30{\cdot}2°$

At Q, Snell's law becomes

$1{\cdot}6 \sin 30{\cdot}2° = 1{\cdot}2 \sin i_2$

$i_2 = 42{\cdot}12°$

Substituting for $r_1$, $i_2$, and $r_2$ in equation (i) gives

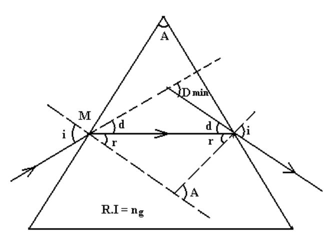
$D = (53° - 36{\cdot}8°) + (42{\cdot}12° - 30{\cdot}2°)$

$\mathbf{D = 28{\cdot}12°}$

### MINIMUM DEVIATION OF LIGHT BY A PRISM

.At minimum deviation, light passes **symmetrically** through the prism. That is to say, the angle of incidence is equal to the angle of emergence.

Consider a ray on one face of the prism at an angle $\mathbf{i_1}$and leaves it at an angle $\mathbf{i_2}$ to the normal as shown




For minimum deviation, $i_1 = i_2 = \mathbf{i}$ and $r_1 = r_2 = \mathbf{r}$.

From the diagram, $D_{min} = d + d$

$D_{min} = 2d$ where $d = i - r$

$\Rightarrow$ $D_{min} = 2i - 2r$--------------------------------(a)

More over , $r + r = A$.

$\Rightarrow$ $2r = A$ **OR** $r = A/2$ ------------------------(b)

Combining equation (a) and (b) gives.

$D_{min} = 2i - A$

$\Rightarrow \quad i = \frac{D_{min} + A}{2}$ ---------------------------------(c)

At M Snell's law becomes

$n_a \sin i = n_g \sin r$

$\Rightarrow \quad n_g = \frac{n_a \sin i}{\sin r}$ --------------------------------(d)

substituting equation (b) and (c) in (d) gives

$$n_g = n_a \frac{\sin\left(\frac{D_{min} + A}{2}\right)}{\sin A/2} ------------(e)$$

Since $n_a = 1$,

$$\Rightarrow \mathbf{n_g} = \frac{\mathbf{\sin\left(\frac{D_{min} + A}{2}\right)}}{\mathbf{\sin A/2}}$$

NOTE ;

Equation (e) suggests that if the prism was surrounded by a medium of refractive index $n_l$, then at minimum deviation

$$n_g = n_l \frac{\sin\left(\frac{D_{min} + A}{2}\right)}{\sin A/2}$$

**EXAMPLES:**

**1**. Calculate the angle of incidence at minimum deviation for light passing through a Prism of refracting angle **70°** and refractive index of **1·65**.

**Solution**

Using $\mathbf{n_g} = \mathbf{n_a} \frac{\mathbf{\sin\left(\frac{D_{min} + A}{2}\right)}}{\mathbf{\sin A/2}}$ where $n_g = 1\cdot 65,\ A = 70°$ and $n_a = 1$

$$\Rightarrow \quad 1\cdot 65 = \frac{\sin\left(\frac{D_{min} + 70°}{2}\right)}{\sin 70°/2}$$

On solving, $\mathbf{D_{min}} = \mathbf{72\cdot 33°}$

The required angle of incidence $\mathbf{i} = \frac{\mathbf{D_{min} + A}}{\mathbf{2}} = \frac{72\cdot 33° + 70°}{2} = \mathbf{71\cdot 165°}$

**2.**An equilateral glass prism of refractive index **1·5** is completely immersed in a liquid of refractive index **1·3**. if a ray of light passes symmetrically through the prism, calculate the:

(i) angle of deviation of the ray.
(ii) angle of incidence

**ANALYSIS:**

(a) For an equilateral prism, its refracting angle A =60°
(b) If the ray passes through the prism symmetrically, then the angle of deviation is minimum

(i) Using $\mathbf{n_g} = \mathbf{n_l} \dfrac{\mathbf{sin}\left(\dfrac{\mathbf{D_{min}} + \mathbf{A}}{\mathbf{2}}\right)}{\mathbf{sin}\,\mathbf{A}/\mathbf{2}}$ where $n_g = 1\cdot 5,\ A = 60°$ and $n_l = 1\cdot 3$

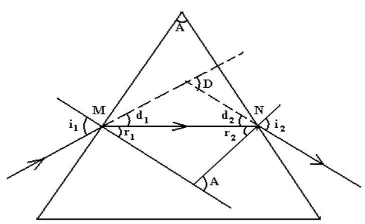
$$\Rightarrow \quad 1\cdot 5 = 1\cdot 3\, \frac{\sin\left(\dfrac{D_{min} + 60°}{2}\right)}{\sin 60°/2}$$

On solving, $\mathbf{D_{min}} = \mathbf{10\cdot 47°}$

(ii) The required angle of incidence $\mathbf{i} = \dfrac{\mathbf{D_{min}} + \mathbf{A}}{\mathbf{2}} = \dfrac{10\cdot 47° + 60°}{2} = \mathbf{35\cdot 235°}$

**DEVIATION OF LIGHT BY A SMALL ANGLE PRISM**

The small refracting angles of this prism causes the angle $\mathbf{i_1, r_1, r_2}$ and $\mathbf{i_2}$ to be small such that $\mathbf{\sin i_1 \approx i_1}$, $\mathbf{\sin r_1 \approx r_1}$ $\mathbf{\sin r_2 \approx r_2}$ and $\mathbf{\sin i_2 \approx i_2}$.




From the diagram, $D = d_1 + d_2$
but $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$
$\Rightarrow \quad D = (i_1 - r_1) + (i_2 - r_2)$
On simplifying $D = i_1 + i_2 - (r_1 + r_2)$
but $r_1 + r_2 = A$
$\therefore \quad D = i_1 + i_2 - A$----------------------(a)
At M Snell's law becomes.
$n_a \sin i_1 = n \sin r_1$
For small angles this gives $i_1 = nr_1$ ---------------------------------(b)
Similarly at N Snell's law becomes $i_2 = nr_2$ ------------------------------(c)
Substituting equation (b) and (c) in (a) gives
$D = nr_1 + nr_2 - A$
$D = n(r_1 + r_2) - A$
but $r_1 + r_2 = A$ $\Rightarrow$ D = nA – A

$\therefore \quad \mathbf{D = (n - 1)A}$

- ∴ **The deviation produced by a small angle prism is independent of the magnitude of the small angle of incidence on the prism. (ie**: All rays entering a small-angle prism at small angles of incidence suffer the same deviation**)**

NOTE:

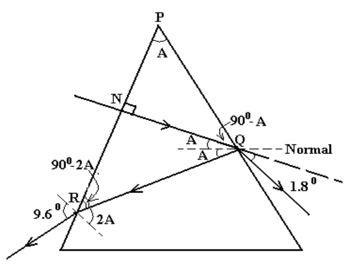
The result **D = (n – 1)A** will later be used in developing lens theory.

**EXAMPLES:**

A ray of light that falls normally upon the first face of a glass prism of a small refracting angle under goes a partial refraction and reflection at the second face of the prism. The refracted ray is deviated through an angle **1·8°** and the reflected ray makes an angle of **9·6°** with the incident ray after emerging from the prism through its first face. Calculate the refracting angle of the prism and its refractive index of the glass material.

Solution

Let **A** be the required refracting angle of the prism as shown




Consider the deviation suffered by the incident light

**D = (n – 1) A** where D = 1·8°

⇒ 1·8° = (n – 1) A --------------------------------------(i)

From ΔPQN, ∠PQN = 90°- A

⇒ At Q, the angle of incidence = A

From ΔNQR, ∠QRN = 90°- 2A

⇒ At R, the angle of incidence = 2A

∴At R, Snell's becomes $n_a$ sin 9·6° = n sin 2A

For small angles, **sin 9·6° ≈ 9·6° and sin A ≈ 2A**

⇒ 9·6° = 2nA ---------------------------------------------(ii)

Equation (i) ÷ Equation (ii) gives

$$\frac{1\cdot 8^\circ}{9\cdot 6^\circ} = \frac{(n-1)A}{2nA}$$

⇒ 3·6°n = 9·6° (n – 1)

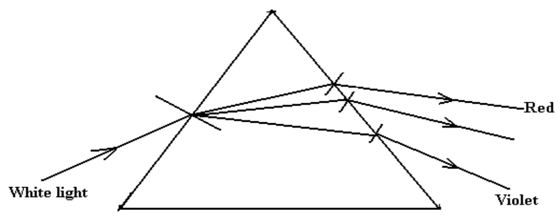
Thus **n = 1·6**

Equation (i) now becomes1·8° = (1·6– 1) A

$\therefore$ **A = 3°**

**DISPERSION OF WHITE LIGHT BY A TRANSPARENT MEDIUM**

Dispersion of whit light is the separation of white light in to its component colours by a transparent medium due to their speed differences in the medium.



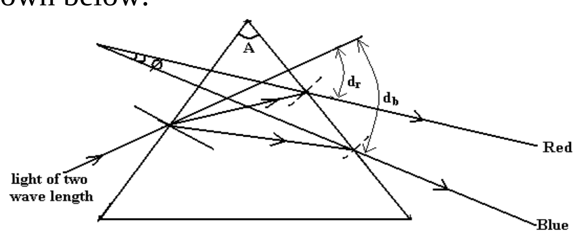

When white light falls on a transparent medium, its different component colours travel with different speeds through the medium. They are there fore deviated by different amounts on refraction at the surface of the medium and hence dispersion.

NOTE :

(i) White light is a mixture of various colours. This is called the spectrum of white light.

(ii) The spectrum of white light consists of red, orange, yellow, green ,blue, indigo and violet light bands. On refraction, violet is the most refracted colour away from the normal ( violet is the most deviated colour ) while red is least deviated

(iii)When light of two wavelengths say red and blue light is incident at a small angle on a small angle prism of refracting angle A having refractive indices of $n_r$ and $n_b$. for the two wave lengths respectively, then the two wave lengths are deviated as shown below.




The deviation of red and blue light is given by $d_r = (n_r - 1)A$

$d_b = (n_b - 1)A$.

The quantity $\phi = d_b - d_r$ is called the **Angular separation (Angular dispersion )** produced by the prism. $\Rightarrow \phi = (n_b - 1)$

$A - (n_r - 1)A$

on simplifying $\boldsymbol{\phi = (n_b - n_r) A}$

**EXAMPLES:**

**1.**Light of two wave length is incident at a small angle on a thin prism of refracting angle 5° and refractive index of 1·52 and 1·48 for the two wave lengths. Find the angular separation of the two wave lengths after refraction by the prism.

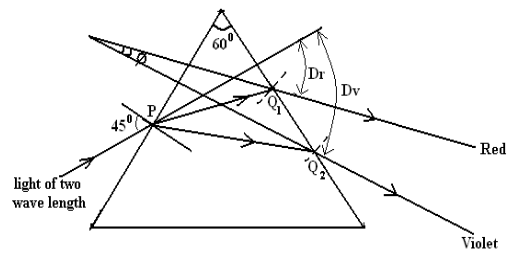
**Solution**

For a small prism, Angular separation $\phi = (n_1 - n_2) A$

$$\phi = (1{\cdot}52 - 1{\cdot}48) \times 5°$$

$$\Rightarrow \phi = \mathbf{0{\cdot}2°}$$

**2.** A glass prism with refracting angle 60° has a refractive index of 1·64 for red light and 1·66 for violet light. Calculate the angular separation of the red and violet rays which emerge from the prism when a ray of white light is incident on the prism at an angle of 45°




**Case I**:

Consider the deviation suffered by red light

At **P,** Snell's law becomes .

$$n_a \sin 45° = 1{\cdot}64 \sin r_1$$

$$\therefore \quad r_1 = 25{\cdot}54°$$

$$\text{But } r_1 + r_2 = 60°$$

$$\Rightarrow \quad r_2 = 60° - r_1$$

$$= 60° - 25{\cdot}54°$$

$$\therefore \quad r_2 = 34{\cdot}46°$$

At $\mathbf{Q_1}$, Snell's law becomes

$$n_a \sin i_2 = 1{\cdot}64 \sin 34{\cdot}46°$$

$$\therefore \quad \mathbf{i_2 = 68{\cdot}13°.}$$

**Total Deviation** $\mathbf{D_r = d_2 + d_1}$ where $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

$$\Rightarrow \quad D_r = (45° - 25{\cdot}54°) + (68{\cdot}13° - 34{\cdot}46°)$$

$$\therefore \quad \mathbf{D_r = 53{\cdot}13°.}$$

**Case II**:

Consider the deviation suffered by violet light

At **P,** Snell's law becomes .

$$n_a \sin 45° = 1{\cdot}66 \sin r_1$$

$$\therefore \quad r_1 = 25{\cdot}21°$$

$$\text{But } r_1 + r_2 = 60°$$

$$\Rightarrow \quad r_2 = 60° - r_1$$

$$= 60° - 25{\cdot}21°$$

$$\therefore \quad r_2 = 34{\cdot}79°$$

At **Q**, Snell's law becomes

$$n_a \sin i_2 = 1{\cdot}66 \sin 34{\cdot}79°$$

$$\therefore \quad \mathbf{i_2 = 71{\cdot}28°.}$$

**Total Deviation** $\mathbf{D_v = d_2 + d_1}$ where $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

$$\Rightarrow \quad D_v = (45° - 25{\cdot}21°) + (71{\cdot}28° - 34{\cdot}79°)$$

$\therefore \quad D_v = 56{\cdot}28°.$

**Thus required angular separation** $\phi = D_v - D_r$

$\phi = 56{\cdot}28° - 53{\cdot}13°$

$\Rightarrow \quad \phi = 3{\cdot}15°$

**APPEARANCE OF WHITE LIGHT PLACED IN WATER**

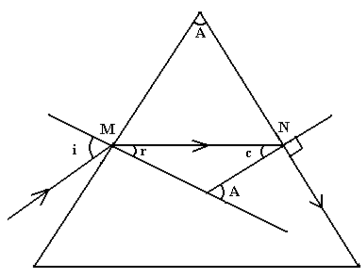
**OBSERVATION:** A coloured spectrum is seen inside the water surface with violet on top and red down.

**EXPLANATION:**

The different component colours of white light travel with different speeds through water. They are there fore deviated by different amounts on refraction at the water surface. Hence different coloured images are formed at different points inside the water surface with a violet coloured image on top.

**GRAZING PROPERTY OF LIGHT RAYS AS APPLIED TO PRISMS.**

If a ray of light is either such that the incident angle or the emergent angle is equal to 90° to the normal of the prism, then the ray is said to graze the refracting surface of the prism. Consider a ray of light incident at an angle i on a glass prism of refracting angle A situated in air with the emergent light grazing the other refracting surface of the prism as shown.




From the diagram, r + c = A

$\therefore$ r = A – c----------------------------------(a)

At M Snell's law becomes

$n_a \sin i = n_g \sin r$----------------------------------(b)

Substituting equation (a) in (b) gives

$\text{Sin } i = n_g \sin (A - c)$

$\Rightarrow \sin i = ng (\sin A \cos C - \sin C \cos A)$-------------------(c)

At N, Snell's law becomes

$n_g \sin c = n_a \sin 90°.$

$\therefore \sin c = \frac{1}{n_g}$

$$\Rightarrow \cos c = \sqrt{1 - \sin^2 c} = \sqrt{1 - \frac{1}{n_g^2}} = \frac{\sqrt{n_g^2 - 1}}{n_g}$$

Substituting sin c and cos c in equation c gives

$$\sin i = n_g\left(\sin A\times\frac{\sqrt{n_g^2-1}}{n_g}-\frac{1}{n_g}\cos A\right)$$

On simplifying we have

$$\sqrt{n_g^2-1}=\frac{\sin i+\cos A}{\sin A}$$

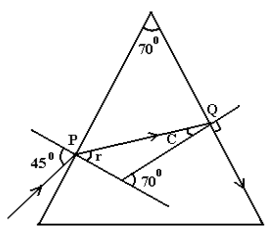
Squaring both sides and simplifying for $n_g$ gives

$$\mathbf{n_g=\sqrt{1+\left(\frac{\sin i+\cos A}{\sin A}\right)^2}}$$

Knowing the angles i and A, the refractive index $n_g$ of a material of a prism can be determined.

EXAMPLES

**1**.Monochromatic light is incident at an angle of **45°** on a glass prism of refracting angle **70°** in air. The emergent light grazes the other refracting surface of the prism. Find the refractive index of the glass material.




At P, Snell's law becomes

$n_a \sin 45° = n_g \sin r$ --------------------------------(a)

From the diagram, r + c = 70°

$\Rightarrow$ r = 70° – c -------------------------------(b)

Substituting equation (b) in (a) gives

Sin 45° = $n_g$ sin ( 70° – c ) -----------------------------------(c)

At Q, Snell's law becomes

$n_g \sin c = n_a \sin 90°$

$$\Rightarrow n_g=\frac{1}{\sin c}-------------(d)$$

Substituting equation (d) in (c) gives

$$\sin 45^o=\frac{\sin(70^o-c)}{\sin c}$$

sin 45° sin c = sin 70° cos c – sin c cos 70°

$\Rightarrow$ ( sin 45° + cos 70° ) sin c = sin 70° cos c

Dividing cos c through out gives

$$\tan c = \frac{\sin 70^o}{\sin 45^o + \cos 70^o}$$

$$\Rightarrow c = 41{\cdot}9^o$$

Equation (d) now becomes $n_g = \frac{1}{\sin 41{\cdot}9^o}$

$$\therefore n_g = 1{\cdot}497$$

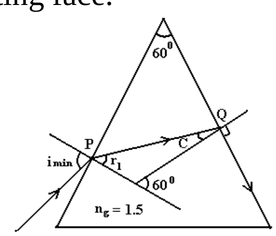
NOTE

For grazing condition, you may as well use the relation $\mathbf{n_g} = \sqrt{\mathbf{1} + \left(\frac{\mathbf{\sin i + \cos A}}{\mathbf{\sin A}}\right)^2}$

$$\Rightarrow n_g = \sqrt{1 + \left(\frac{\sin 45^o + \cos 70^o}{\sin 70^o}\right)^2}$$

$$\therefore \ n_g = 1{\cdot}498$$

2..A ray of light is incident on one refracting face of a prism of refractive index 1·5 and refracting angle 60°. Calculate the minimum angle of incidence for the ray to emerge through the second refracting face.




ANALYSIS

for minimum angle of incidence, the emergent ray grazes the second refracting face.

At **Q**, Snell's law becomes

$1{\cdot}5 \sin c = n_a \sin 90°$

$\Rightarrow \ c = 41{\cdot}8°$

But $r + c = 60°$

$\Rightarrow \ r = 60° - c$

$= 60° - 41{\cdot}8°$

$\therefore \ r = 18{\cdot}2°$

At **P**, Snell's law becomes

$1{\cdot}5 \sin 18{\cdot}2° = n_a \sin i_{min}$

$\therefore i_{min} = 27{\cdot}9°$

NOTE

For grazing condition, you may as well use the relation $\mathbf{n_g} = \sqrt{\mathbf{1} + \left(\frac{\mathbf{\sin i + \cos A}}{\mathbf{\sin A}}\right)^2}$

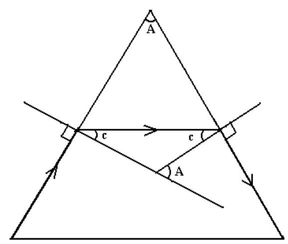
$$\Rightarrow 1\cdot5 = \sqrt{1 + \left(\frac{\sin i_{min} + \cos 60^o}{\sin 60^o}\right)^2}$$

On simplifying, $\mathbf{i_{min}}$ **= 27·9°**

**LIMITING ANGLE OF THE PRISM**

This is the maximum refracting angle of the prism for which he emergent ray grazes the second refracting surface.

Suppose the incident ray grazes the first refracting surface then the limiting angle A is given by A = 2c where **c** is the critical angle of the glass air interface as shown.




**EXERCISE:**

**1.** (i) Obtain an expression relating the deviation of a ray of light by the prism to the refracting angle and the angles of incidence and emergence.

(ii) The deviation of a ray of light incident on the first face of a **60°** glass prism at an angle of **45°** is **40°.** Calculate the angle of emergence of a ray on the second face of the prism.

[ Ans $\mathbf{i_2}$ = 65° ]

(iii) A prism of refractive index **1·64** is immersed in a liquid of refractive index **1·4**. A ray of light is incident on one face of the prism at an angle of **40°.** If the ray emerges at an angle of **29°,** determine the angle of the prism.

**[Answer: 57·7°]**

**2. (**i) For a ray of light passing through the prism, what is the condition for minimum deviation to occur?

(ii) Derive an expression for the refractive index of a prism in terms of the refracting angle, **A**, and the angle of minimum deviation **D**.

(iii) A glass prism of refractive index n and refracting angle ,**A**, is completely immersed in a liquid of refractive index $\mathbf{n_i}$. If a ray of light that passes symmetrically through the prism is deviated through an angle **ϕ,** Show that

$$\frac{\mathbf{n_i}}{\mathbf{n}} = \frac{\mathbf{\sin A/2}}{\mathbf{\sin\left(\frac{\varphi + A}{2}\right)}}$$

3.(a) A glass prism with refracting angle **60°** is made of glass whose refractive indices for red and violet light are respectively **1·514** and **1·530.** A ray of white light is set incident on the prism to give a minimum deviation for red light.

Determine the:

**(i)** angle of incidence of the light on the prism.
**(ii)** angle of emergence of the violet light.
**(iii)** angular width of the spectrum.

(b) A certain prism is found to produce a minimum deviation of **51°.** While it produces a deviation of **62·8°** for a ray of light incident on its first face at an angle of **40·1°** and emerges through its second face at an angle of **82·7°.**
Determine the:

**(i)** refracting angle of the prism.
**(ii)** angle of incidence at minimum deviation.
**(iii)** refractive index of the material of the prism.

**[ Ans (i) 60° (ii) 55·5° (iii) 1·648 ]**

**4. (i)** A ray of monochromatic light is incident at a small angle of incidence on a small angle prism in air. Obtain the expression **D = ( n – 1 )A** for the deviation of light by the prism.

**(ii)** A glass prism of small angle ,**A**, and refractive index $\mathbf{n_g}$ and is completely immersed in a liquid of refractive index $\mathbf{n_L}$. Show that a ray of light passing through the prism at a small angle of incidence suffers a deviation given by

$$\mathbf{D} = \left(\frac{\mathbf{n_g}}{\mathbf{n_L}} - \mathbf{1}\right)\mathbf{A}$$

**5.** Explain why white light is dispersed by a transparent medium.

**6.** Light of two wave length is incident at a small angle on a thin prism of refracting angle 5° and refractive index of 1·52 and 1·48 for the two wave lengths. find the angular separation of the two wave lengths after refraction by the prism.
[ Ans $\phi$ = 0·2° ]

**7.**A point source of white light is placed at the bottom of a water tank in a dark room. The light from the source is observed obliquely at the water surface. Explain what is observed.

**8.** Monochromatic light is incident at an angle $\phi$ on a glass prism of refracting angle ,A, situated in air. If the emergent light grazes the other refracting surface of the prism, Show that the refractive index, $n_g$, of the prism material is given by

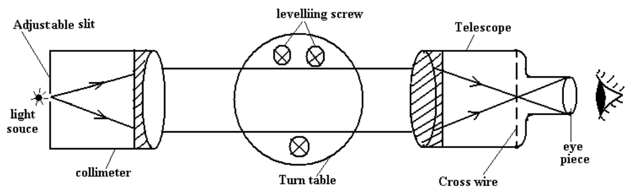
$$\mathbf{n_g} = \sqrt{\mathbf{1} + \left(\frac{\mathbf{\sin i + \cos A}}{\mathbf{\sin A}}\right)^2}$$

**9.** A ray of light is incident at angle of 30° on a prism of refractive index 1·5 .calculate the limiting angle of the prism such that the ray does not emerge when it meets the second face.
[ Ans A = 61·3° ]

**A SPECTROMETER**

It is an instrument used to measure accurate determination of deviation of a parallel beam of light which has passed through a prism. This provides a mean of studying optical spectra and measurement of refractive indices of glass in form of a prism.

It consists of a collimator, a telescope, and a turn table on which the prism is placed as shown.

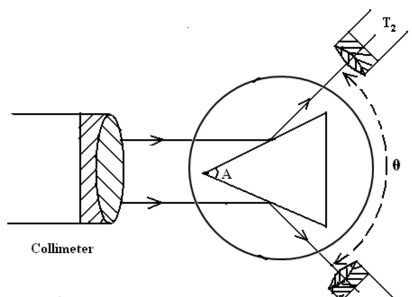



Before the spectrometer is put in to use, 3 adjustments must be made onto it and these include,

(i) The collimator is adjusted to produce parallel rays of light.
(ii) The turntable is leveled.
(iii)The telescope is adjusted to receive light from the collimator on its cross wire.

### MEASUREMENT OF THE REFRACTING ANGLE "A" OF THE PRISM

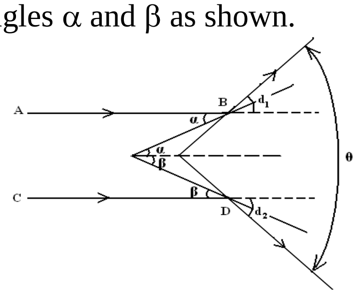
The collimator is adjusted to produce parallel rays of light.
The turn table is leveled.
The telescope is adjusted to receive light fro m the collimator on its cross wire. The prism is placed on the turn table with its refracting angle facing the collimator as shown.




With the table fixed, the telescope is moved to position $T_1$ to receive the light from the collimator on its cross wire. This position $T_1$ is noted and the telescope is turned n to a new position $T_2$ to receive light on its cross wire. The angle θ between $T_1$ and $T_2$ Is measured. The prism angle A is given by $\mathbf{A = \frac{1}{2}\theta}$

### PROOF OF THE RELATION

Consider a parallel beam of light incident on to a prism of refracting angle A making glancing angles α and β as shown.




From the geometry, $\alpha + \beta = A$------------------------------.(i).

Deviation $d_1$ of ray AB = $2\alpha$

Deviation $d_2$ of ray CD = $2\beta$.

Total deviation $\theta = d_1 + d_2$

$= 2\alpha + 2\beta$

$= 2(\alpha + \beta)$--------------------------(ii)

Combining equation (i) and (ii) gives

$$\boldsymbol{\theta = 2A.}$$

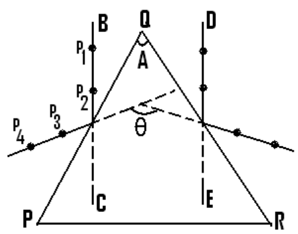
NOTE :

It is now clear from the geometry that the angle θ turned through in moving the telescope from $T_1$ to $T_2$ is given by $\theta = 2A$

Thus $A = \frac{1}{2}\theta$.

**METHOD 2: USING OPTICAL PINS**

A white paper is stuck to the soft board using top-headed pins. Two parallel line are AB and DC are drawn on the paper and the prism is placed with its apex as shown.




Two optical pins $P_1$and $P_2$ are placed along AB and pins $P_3$ and $P_4$ are placed such that they appear to be in line with the images of $P_1$and $P_2$ as seen by reflection from face PQ .The procedure is repeated for face QR. The prism is removed and angle $\theta$ is measured.The required refracting angle $A = \theta/2$

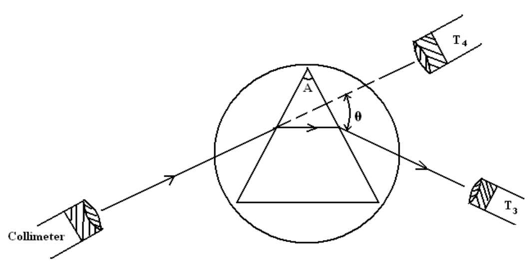
**MEASUREMENT OF MINIMUM DEVIATION "$D_{min}$" OF THE PRISM**

The collimator is adjusted to produce parallel rays of light.

The turn table is leveled.

The telescope is adjusted to receive light fro m the collimator on its cross wire

The prism is placed on a turn table with its refracting angle facing away from the collimator as shown.




The telescope is turned in the direction of the base of the prism until light can be seen. With light kept in view, both the telescope and the table are turned until light moves in the opposite direction. Position $T_3$ of the telescope is noted. The table is then fixed and the prism is removed so that the telescope is turned to a new position $T_4$

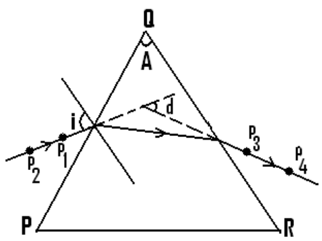
to receive the un deviated light. The angle between $T_3$ and $T_4$ is determined and this is the angle of minimum deviation $\mathbf{D_{min}}$.

NOTE:

(i) Position $T_3$ is noted because, in the position of minimum deviation light viewed through the telescope moves in the opposite direction.

(ii)The refracting index of a glass prism of known refracting angle A can be determined using a spectrometer from the relation $n = \dfrac{\sin\left(\frac{D_{min} + A}{2}\right)}{\sin \frac{A}{2}}$

**METHOD 2: USING OPTICAL PINS**




Two optical pins $\mathbf{P_1}$and $\mathbf{P_2}$ are placed along the lines that make different angles of incidence **i.** Pins $\mathbf{P_3}$ and $\mathbf{P_4}$ are placed such that they appear to be in line with the images of $\mathbf{P_1}$ and $\mathbf{P_2}$ as seen through the prism. The angles of deviation **d** are measured for different angles of incidence. A graph of **d** against **i** is plotted to give a curve whose angle of deviation at its turning point is the angle of minimum deviation $\mathbf{D_{min}}$ of the prism.

**USES OF A GLASS PRISM.**

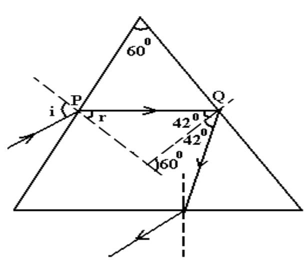
1. They enable the refractive index of a glass material to be measured accurately.
2. They are used in the dispersion of light emitted by glowing objects.
3. They are used as reflecting surfaces with minimal energy loss.
4. They are used in prism binoculars.

## More worked out examples

1.A ray of monochromatic light is incident on one face of a glass prism of refracting angle 60° and is totally internally reflected at the next face.

(i) Draw a diagram to show the path of light through the prism.

(ii) Calculate the angle of incidence at the first face of the prism if its refractive index is 1·53 and the angle of incidence at the second face is 42°.



From the diagram, r + 42° = 60°

$\therefore$ r = 18°

At P, Snell's becomes

$n_a \sin i = 1{\cdot}53 \sin 18°$

$\therefore \; i = 28{\cdot}2°$

NOTE

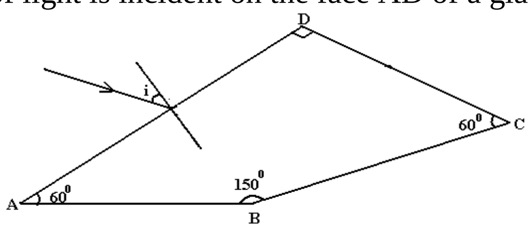
For $n_g$ =1·53, then the critical angle c for the above glass material is given by the relation

$$\sin c = \frac{1}{n_g} = \frac{1}{1\cdot53}$$

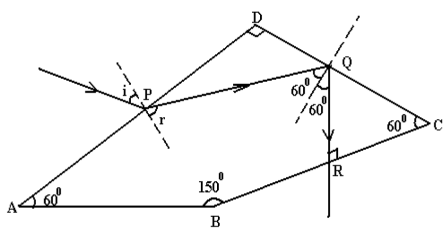
$$\therefore \; c = 40\cdot8^{o}$$

Thus total internal reflection occurs at **Q** since the angle of incidence is greater than the critical angle **c**

7. A ray of light is incident on the face AD of a glass block of refractive index 1·52 as shown.




If the ray emerges normally through face BC after total internal reflection, Calculate the angle of incidence, **i**.




ANALYSIS

(i) Its after a total internal reflection at Q that the ray emerges through face DC

(ii) At R, there is no refraction. There fore Snell's law does not hold at this point.

From $\Delta RCQ$, $\angle RQC + 60° + 90° = 180°$

$\therefore \angle RQC = 30°$

$\Rightarrow$ At Q, the angle of reflection = 60°

Hence at Q, the angle of incidence = 60°

Solving $\Delta QDP$ gives $\angle QPD = 60°$

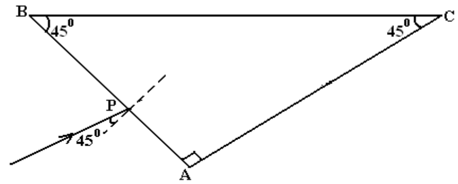
Hence at P, the angle of refraction **r** = 30°

$\Rightarrow$ At P, Snell's law becomes

$n_a \sin i = 1{\cdot}52 \sin 30°$

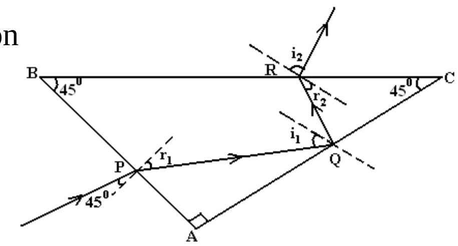
$\therefore \; i = 49{\cdot}5°$

8. A ray of light is incident at 45° on a glass prism of refractive index 1·5 as shown.




Calculate the angle of emergence and sketch the ray diagram.

Solution




At P, Snell's becomes $n_a \sin 45° = 1{\cdot}5 \sin r$

$\therefore \quad r_1 = 28{\cdot}1°$

At P, $\angle APQ + r_1 = 90°$ where $r_1 = 28{\cdot}1°$

$\Rightarrow \quad \boldsymbol{\angle APQ = 61{\cdot}9°}$

From $\Delta APQ$, $\angle PQA + 61{\cdot}9° + 90° = 180°$

$\therefore \quad \angle PQA = 28{\cdot}1°$

$\Rightarrow$ At Q, the angle of incidence **i = 61·9°**

**Tesiting for total internal reflection at Q using the relation** $\mathbf{\sin c = \frac{1}{n_g} = \frac{1}{1{\cdot}5}}$ **gives** $\mathbf{c = 41{\cdot}8°}$

Thus light is totally reflected at **Q** since **i** > **c**.

$\Rightarrow \quad \boldsymbol{\angle PQA = \angle RQC = 28{\cdot}1°}$

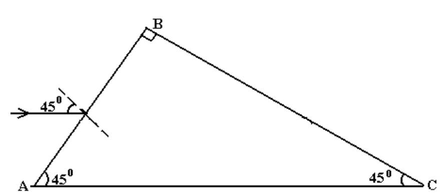
From $\Delta RQC$, $28{\cdot}1° + 45° + 90° + r_2 = 180°$

$r_2 = 16{\cdot}9°$

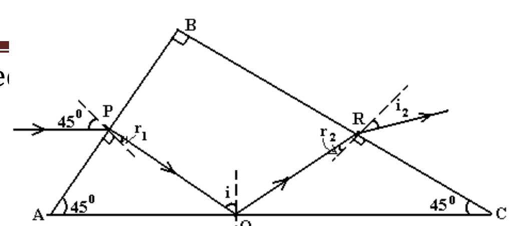
At R, Snell's becomes $1{\cdot}5 \sin 16{\cdot}9° = n_a \sin i_2$

**Thus** $\mathbf{i_2 = 25{\cdot}15°}$

9. A ray of light is incident at 45° on a glass prism of refractive index 1·5 as shown.




Calculate the angle of emergence and sketch the ray diagram.
Solution



At P, Snell's becomes $n_a \sin 45° = 1{\cdot}5 \sin r$

$\therefore \quad r_1 = 28{\cdot}1°$

From $\Delta APQ$, $\angle PQA + 45° + 90° + r_1 = 180°$ where $r_1 = 28{\cdot}1°$

$\therefore \quad \angle PQA = 16{\cdot}9°$

$\Rightarrow$ At Q, the angle of incidence **i = 73·1°**

**Tesiting for total internal reflection at Q using the relation** $\mathbf{\sin c = \frac{1}{n_g} = \frac{1}{1\cdot 5}}$ **gives** $\mathbf{c = 41\cdot 8°}$

Thus light is totally reflected at **Q** since **i > c**.

$\Rightarrow \ \angle \mathbf{PQA} = \angle \mathbf{RQC} = \mathbf{16{\cdot}9°}$

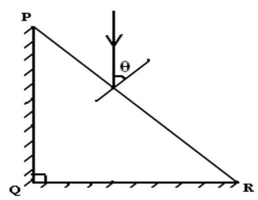
From $\Delta RQC$, $16{\cdot}9° + 45° + 90° + r_2 = 180°$

$r_2 = 28{\cdot}1°$

At R, Snell's becomes $1{\cdot}5 \sin 28{\cdot}1° = n_a \sin i_2$

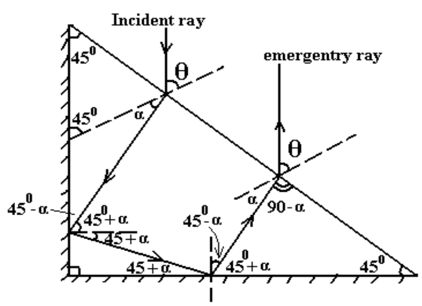
**Thus** $\mathbf{i_2 = 45°}$

10.The diagram in the figure below shows a cross section of an isosceles right angled prism sides PQ and QR are coated with a reflecting substance. A ray of light is incident on PR at an angle $\theta$ as shown




(i) Draw a diagram to show the path of light through the prism.

(ii) Show that the ray leaving the prism is parallel to the incident ray.




From geometry of the figure above the angle of emergence at D is the same as the incident angle $\theta$ at A. Hence the emergent ray is parallel to the incident ray.

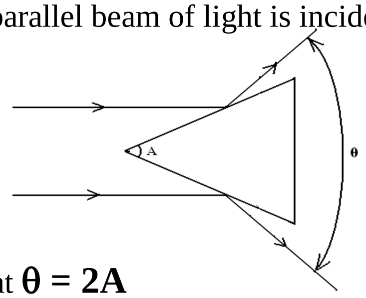
**EXERCISE**

**1.** Draw a labeled diagram of a spectrometer and State the necessary adjustments that must be made on to it before put in to use.

**2.** Describe how the refracting angle of the prism can be measured using a spectrometer.

**3.** You are provided with pins, a white sheet of paper, a drawing board and a triangular prism. Describe how you would determine the refracting angle **A** of the prism

5. A parallel beam of light is incident on to a prism of refracting angle, **A**, as shown




Show that $\theta = 2A$

**5**.Describe how the minimum deviation, **D**, of a ray of light passing through a glass prism can be measured using a spectrometer.

**6**.You are provided with pins, a white sheet of paper, a drawing board and a triangular prism. Describe how you would determine the angle of minimum deviation, **D,** of a ray of light passing through a glass prism.

**7**. Describe how the refractive index of a material of a glass prism of known refracting angle can be determined using a spectrometer.

**8.** Describe briefly two uses of glass prisms

## REFRACTION THROUGH LENSES

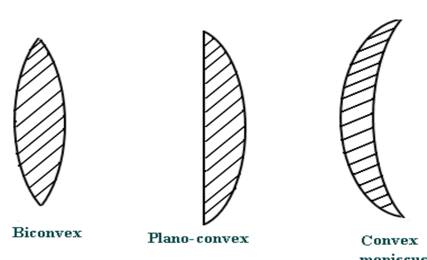
A lens is a piece glass bounced by one or two spherical surfaces.

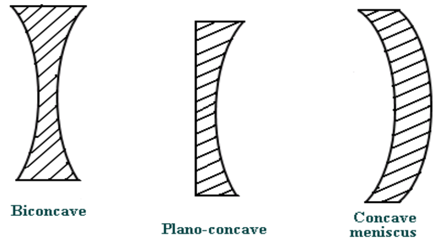
Lenses are of two types namely; **(i)**

**Convex (Converging) lens:** It is a lens which is thicker in the middle than at the edges.

**(ii) Concave (Diverging) lens:** It is a lens which is thinner in the middle than at the edges.

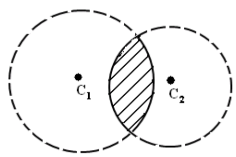
Illustrated below are some of the common types of lenses.






**Definitions:**

**1. Centres of curvature of a lens:** These are centres of the spheres of which the lens surfaces form parts.




Points $C_1$ and $C_2$ are the centers of curvature of the lens surfaces.

**2. Radii of curvature of a lens:** These are distances from the centers to the surfaces of the spheres of which the lens surfaces form parts.

**3. Principal axis of a lens:** This is the line joining the centers of curvature of the two surfaces of the lens.

**4**. **Optical centre of the lens:** This is the mid-point of the lens surface through which rays incident on the lens pass undeviated.

**5. Paraxial rays:** These are rays close to the principle axis and make small angles with the lens axis.

**6. (i) Principal focus "F" of a convex lens:** it is a point on the principal axis where paraxial rays incident on the lens and parallel to the principal axis converge after refraction by the lens.

**(ii) Principal focus "F" of a concave lens:** it is a point on the principal axis where paraxial rays incident on the lens and parallel to the principal axis appear to diverge from after refraction by the lens.

**7.(i) Focal length "f" of a convex lens:** it is the distance from the optical centre of the lens to the point where paraxial rays incident and parallel to the principal axis converge after refraction by the lens.

(**ii) Focal length "f" of a concave lens:** it is the distance from the optical centre of the lens to the point where paraxial rays incident and parallel to the principal axis appear to diverge from after refraction by the lens.

**GEOMETRICAL RULES FOR THE CONSTRUCTION OF RAY DAIGRAMS**

The following is a set of rules for easy location of the images formed by lenses:

**(i)** A ray parallel to the principal axis after refraction passes through the principal focus or appears to diverge from it.

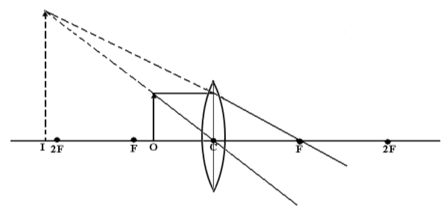
**(ii)** A ray through the principal focus is refracted parallel to the principal axis.
**(iii)** A ray through the optical centre continues straight or undeviated.

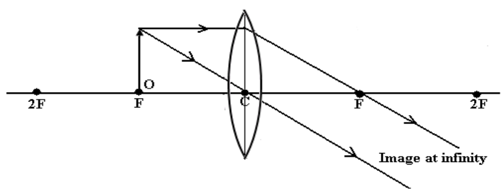
**NOTE**:

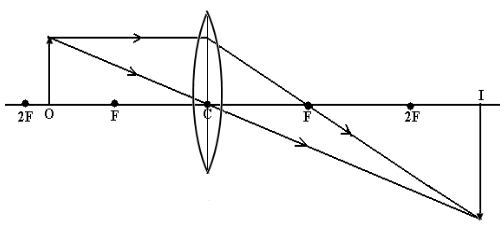
**(i)** The normals to the lens surfaces must pass through its centres of curvature.
**(ii)** The image position can be located by the intersection of two refracted rays initially coming from the object.

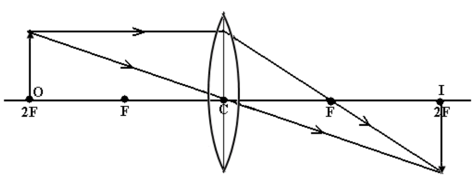
**IMAGES FORMED BY A CONVEX LENS**

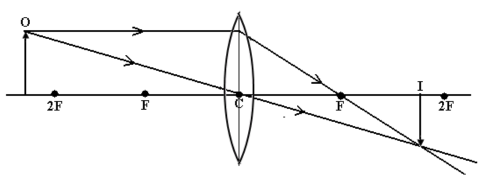
The nature of the image formed by a convex lens is either real or virtual depending on the object distance from the lens as shown below;

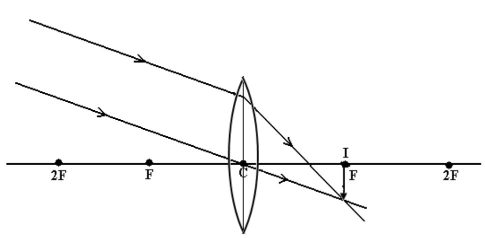


**NOTE;**

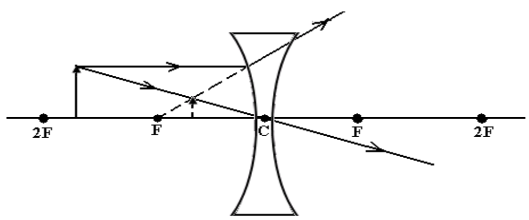
Generally the image of an object in a convex lens is virtual only when the object is nearer to the lens than its focus.

**USES OF CONVEX LENSES**

**(i)** They are used in spectacles for long-sighted people
**(ii)** They are used in cameras.
**(iii)** They are used in projectors.
**(iv)**They are used in microscopes.
**(v)** They are used in astronomical telescopes.

**IMAGES FORMED BY A CONCAVE LENS**

The image of an object in a convex lens is erect, virtual, and diminished in size no matter where the object is situated as shown below

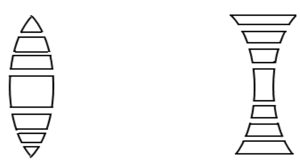



**USES OF CONCAVE LENSES**

**(i)** They are used in spectacles for short-sighted people
**(ii)** They are used in Galilean telescopes.

**LENSES COMPARED WITH PRISMS**

A thin lens may be regarded as being made up of several portions of small-angle prisms placed together as shown:



All paraxial rays incident on the lens at a single point suffer a similar small deviation
**D = (n – 1) A** produced by a prism of small-angle **A** regarded to have replaced the lens.

## LENSES AND SIGN CONVENTION

**Sign conventions** are rules followed in assigning signs to focal length, radius of curvature, object and image distances.

In order to obtain a formula which holds for both concave and convex lenses, a sign rule or convention must be obeyed and the following shall be adopted.
**(i)** Distances of real objects and images are positive.

**(ii)** Distances of virtual objects and images are negative.

**NOTE:**

A convex lens has a positive focal length while a concave lens has a negative focal length.

**THE THIN LENS FORMULAR**

Consider in each case a paraxial ray incident on the same lens at a small height **h** above the principal axis as shown:

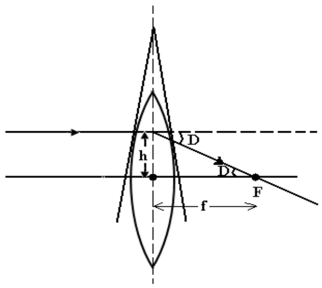


Fig (i)

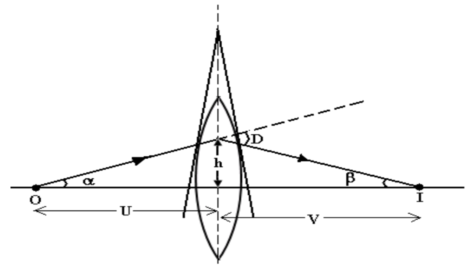


Fig (ii)

From Fig (i), the ray is converged to the focal point **F** to suffer a small deviation **D**

where $\mathbf{D} \approx \tan \mathrm{D} = \dfrac{\mathbf{h}}{\mathbf{f}}$ ---------------------------------- (i)

From Fig (ii), the ray from a point object **O** suffers the same small deviation **D** to give rise to a point object **I**.

From geometry, $\mathrm{D} = \alpha + \beta$ where $\boldsymbol{\alpha} \approx \tan \alpha = \dfrac{\mathbf{h}}{\mathbf{u}}$ and $\boldsymbol{\beta} \approx \tan \beta = \dfrac{\mathbf{h}}{\mathbf{v}}$

$\Rightarrow$ $\mathbf{D} = \dfrac{\mathbf{h}}{\mathbf{u}} + \dfrac{\mathbf{h}}{\mathbf{v}}$ ----------------------------- (ii)

Equating equations (i) and (ii) gives

$$\frac{h}{f} = \frac{h}{u} + \frac{h}{v}$$

**Thus** $$\frac{\mathbf{1}}{\mathbf{f}} = \frac{\mathbf{1}}{\mathbf{u}} + \frac{\mathbf{1}}{\mathbf{v}}$$

This is the general lens equation. If the sign conversion is used for **u, v** and **f** the equation holds for both converging and diverging lenses.

**NOTE:**

The same result above can be derived with the aid of a concave lens having light ray paths shown below:

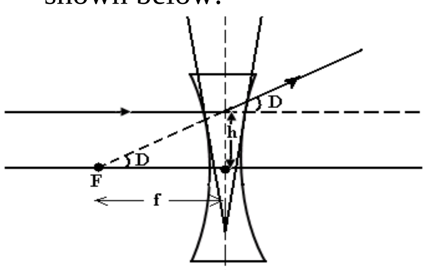


Fig (i)

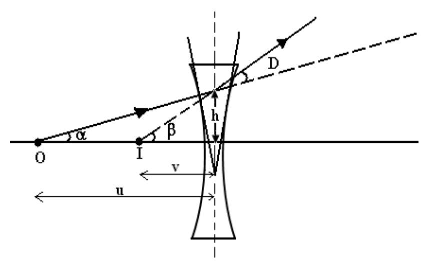


Fig (ii)

**POWER OF A LENS**

This is the reciprocal of the focal length of the lens in meters. It is measured in dioptres.

$$\text{Hence power of a lens} = \frac{1}{\text{focal length } \mathbf{(in\,meters)}}$$

**NOTE:**

The power of a converging lens is positive while that of a diverging lens is negative.

**EXAMPLE:**

**1.** Calculate the power of a converging lens of focal length **25cm**.
**Solution**:

$$\text{Power of a converging lens} = \frac{1}{25\times10^{-2}\ \text{(in meters)}} = \mathbf{4dioptres}$$

**2.** Calculate the power of a diverging lens of focal length **20cm**.
**Solution**:

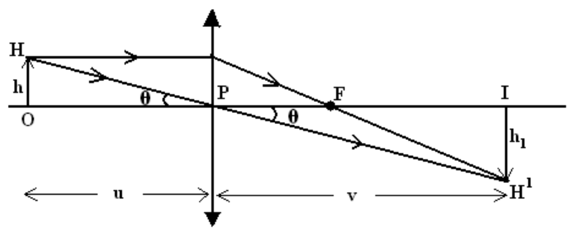
$$\text{Power of a diverging lens} = \frac{1}{-20\times10^{-2}\ \text{(in meters)}} = \mathbf{-5dioptres}$$

**FORMULA FOR MAGNIFICATION**

$$\text{Linear magnification, m,} = \frac{\text{Image height}}{\text{Object height}} = \frac{\text{Image distance}}{\text{Object distance}}.$$

**PROOF**

Consider the formation of a real image of height $\mathbf{h_1}$ of an object of height **h** by a lens as shown




From Δ POH, $\tan\theta = \frac{h}{u}$ ------------------- (i)

From Δ PIH[1], $\tan\theta = \frac{h_1}{v}$ ---------------------------.(ii)

Equating equation (i) and (ii) gives.

$$\frac{v}{u} = \frac{h_1}{h}$$

**Thus magnification, m,** $= \mathbf{\frac{v}{u} = \frac{h_1}{h}}$

**NOTE:**

**(i)** No signs need be inserted in the magnification formula. **(ii)** Using the lens formula, a connection relating magnification to the focal length of the lens with either the object distance or the image distance can be established.

**RELATIONSHIP CONNECTING m, v and f**

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Multiplying, v, throughout the expression gives,

$$\frac{v}{f} = \frac{v}{u} + 1$$

**But** $\frac{v}{u} = m$

$$\Rightarrow \frac{v}{f} = m + 1$$

$$\therefore \mathbf{m = \frac{v}{f} - 1}$$

**RELATIONSHIP CONNECTING m, u and f**

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Multiplying, u, throughout the expression gives,

$$\frac{u}{f} = 1 + \frac{u}{v}$$

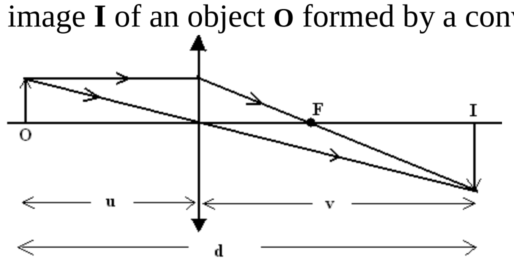
**But** $\frac{u}{v} = \frac{1}{m}$

$$\Rightarrow \frac{u}{f} = 1 + \frac{1}{m}$$

$$\therefore \mathbf{\frac{1}{m} = \frac{u}{f} - 1}$$

**MINIMUM SEPARATION OF AN OBJECT AND ITS REAL IMAGE FORMED BY A CONVEX LENS**

Consider a real image **I** of an object **O** formed by a convex lens of focal length **f** as shown:




Let **d** be the separation of an object and its real image formed by a convex lens

$\Rightarrow$ Image distance **v = d - u** where **u** = object distance

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ gives

$$f = \frac{uv}{u+v} = \frac{u(d-u)}{u+d-u} = \frac{du - u^2}{d}$$

$$\Rightarrow \quad u^2 - du + df = 0$$

Solving for u gives

$$u = \frac{d \pm \sqrt{d^2 - 4df}}{2} \quad \text{------------------(i)}$$

For the solution of u to be real, $d^2 - 4df \geq 0$

$$\Rightarrow \quad d^2 \geq 4df$$

$$\therefore \quad \mathbf{d \geq 4f}$$

**Thus the minimum distance between an object and its real image formed by a convex lens of focal length f is 4f**

**NOTE:**

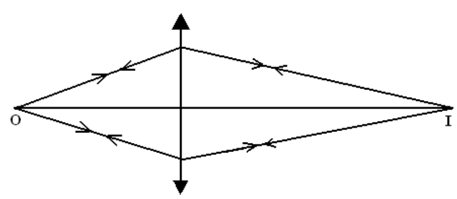
When the separation $\mathbf{d = 4f}$, $\mathbf{u} = \frac{4f \pm 0}{2} = \mathbf{2f}$

$$\Rightarrow \quad \mathbf{v} = d - u = 4f - 2f = \mathbf{2f}$$

**Thus when the separation is 4f, the object and its real image are then equidistant from the lens on opposite sides.**

**LENSES AND REVERSIBILITY OF LIGHT**

Consider the formation of a real image of an object by a convex lens as shown:




Reversibility of light is the behavior of light to travel between an object and its real image along the same path in either direction. This means that an object and its image at these positions are interchangeable.

**NOTE:**

The principle of reversibility of light states that the paths of light rays are reversible. This means that a ray of light can travel from position **1** to **2** and from **2** to **1** along the same path.

**CONJUGATE POINTS**

These are two points where by if an object is placed at one of the points, its image is formed at the other point by the lens.

**NOTE:**

The property of conjugate points is such that an object and its image at these points are interchangeable.

**DISPLACEMENT OF A LENS WHEN OBJECT AND SCREEN ARE FIXED**

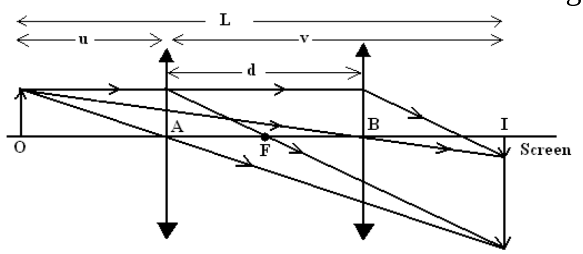
Consider a converging lens placed between an object and the screen. Here two cases of interest are considered.

**CASE I**:

The position of the screen is adjusted until a clear magnified image is obtained on the screen.

**CASE II**:

Keeping the screen fixed in this position at a distance **L** from the object, the lens is displaced through a distance **d** to obtain a clear diminished image on the screen as shown.




**O** and **I** are conjugate points therefore it follows that

**u = OA = BI** and **v = AI = OB.**

From above, OA + AB + BI = L

$\Rightarrow \quad u + d + u = L$

$\therefore \quad 2u = L - d.$

$$u = \frac{L - d}{2} \quad \text{------------------------------------} \text{(i)}$$

Also u + v = L

$\therefore \quad v = L - u$

$$\Rightarrow \quad v = L - \left(\frac{L - d}{2}\right) = \frac{L + d}{2}$$

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ gives

$$f = \frac{uv}{u + v} = \frac{\left(\frac{L - d}{2}\right)\left(\frac{L + d}{2}\right)}{\left(\frac{L - d}{2}\right) + \left(\frac{L + d}{2}\right)}$$

$$\therefore \quad \mathbf{f} = \frac{\mathbf{L^2 - d^2}}{\mathbf{4L}}$$

**Hence if the displacement d of the lens and the distance L between an object and the screen are measured, the focal length f of the lens can be determined.**

**NOTE:**

(i) The magnification $m_1$ produced by the lens in position **A** is

$$m_1 = \frac{AI}{OA} \quad \text{where } AI = v = \frac{L+d}{2} \quad \text{and} \quad OA = u = \frac{L-d}{2}$$

$$m_1 = \frac{\left(\frac{L+d}{2}\right)}{\left(\frac{L-d}{2}\right)}$$

$$\therefore \quad \mathbf{m_1 = \frac{L+d}{L-d}}$$

(ii) The magnification $m_2$ produced by the lens in position **B** is

$$m_2 = \frac{BI}{OB} \quad \text{where } BI = u = \frac{L-d}{2} \quad \text{and} \quad OB = v = \frac{L+d}{2}$$

$$m_2 = \frac{\left(\frac{L-d}{2}\right)}{\left(\frac{L+d}{2}\right)}$$

$$\therefore \quad \mathbf{m_2 = \frac{L-d}{L+d}}$$

(iii) The product of the magnifications produced in the two cases is

$$\mathbf{m_1 \times m_2 = \left(\frac{L+d}{L-d}\right)\left(\frac{L-d}{L+d}\right)}$$

$$\textbf{Thus } \mathbf{m_1 \times m_2 = 1}$$

However the magnification $m_1$ and $m_2$ in terms of the image and the object height are given by

$$m_1 = \frac{h_1}{h} \quad \text{and} \quad m_2 = \frac{h_2}{h}$$

$$\therefore \mathbf{m_1 \times m_2 = \frac{h_1}{h} \times \frac{h_2}{h} = \frac{h_1 h_2}{h^2}}$$

$$\Rightarrow \mathbf{1 = \frac{h_1 h_2}{h^2}}$$

$$\textbf{Thus } \mathbf{h = \sqrt{h_1 h_2}}$$

Consequently the length **h** of an object can easily be found by measuring the lengths of $\mathbf{h_1}$ and $\mathbf{h_2}$ of the images for the two positions of the lens.

This method of measuring **h** is most useful when the object is inaccessible for example

(i) When the width of the slit inside the tube is required. (ii) When the focal length of a thick lens is required.

**CONDITION FOR THE FORMATION OF A REAL IMAGE BY A CONVEX LENS**

(i) The object distance must be greater than the focal length of the lens.

(ii) The distance between the object and the screen must be at least four times the focal length of the lens.

**EXAMPLES:**

**1.** A real image in a converging lens of focal length 15cm is twice as long as the object. Find the image distance from the lens

**Solution**

Using the relatrion $m = \frac{v}{f} - 1$

$\Rightarrow$ $v = (m + 1)f$ where $m = 2$, and $f = 15cm$

$\therefore$ $v = (2 + 1) \times 15$

**v = 45cm**.

**2.** A convex lens of focal length 15cm forms an image three times the height of its object. Find the possible object and corresponding image positions. State the nature of each image.

Let the object distance = u

$\Rightarrow$ The possible image distances = +3u **or** –3u

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

**Case I where the image distance = +3u**

$$\frac{1}{15} = \frac{1}{u} + \frac{1}{3u}$$

$$\frac{1}{15} = \frac{4}{3u}$$

$\therefore$ The object distance u = 20cm

The corresponding image distance = 3u = 3 × 20cm = 60cm

The image is real, magnified, inverted and behind the lens

**Case II where the image distance = –3u**

$$\frac{1}{15} = \frac{1}{u} - \frac{1}{3u}$$

$$\frac{1}{15} = \frac{2}{3u}$$

$\therefore$ The object distance u = 10cm

The corresponding image distance = –3u = –3 × 10cm = –30cm

The image is virtual, magnified, erect and same side with the object

**3**.The magnification of an object in a converging lens is **m**. when the lens is moved a distance **d** towards the object, the magnification becomes $\mathbf{m^1}$ show that the focal length **f** of the lens is given by $\mathbf{f = \frac{dmm^1}{m^1 - m}}$ .

Let u be the object distance before the displacement

$$\Rightarrow \qquad \frac{1}{m} = \frac{u}{f} - 1 \text{ ----------------------- (i)}$$

After the displacement, object distance becomes u - d

$$\Rightarrow \qquad \frac{1}{m^1} = \frac{u-d}{f} - 1 \text{ --------------------- (ii)}$$

Equation (i) - Equation (ii) elliminates u to give

$$\frac{1}{m} - \frac{1}{m^1} = \frac{d}{f}$$

$$\Rightarrow \qquad \mathbf{f = \frac{dmm^1}{m^1 - m}}$$

**4**. Two thin convex lenses **A** and **B** of focal lengths **5cm** and **15cm** respectively are placed coaxially **20cm** apart. If an object is placed **6cm** from **A** on the side remote from **B**,

**(i)** Find the position, nature and magnification of the final image.

**(ii)** Sketch a ray diagram to show the formation of the final image.

Solution

**Consider the action of a convex lens A**

u = 6cm and f =5cm

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ gives

$$v = \frac{f \cdot u}{u - f} = \frac{5 \times 6}{6 - 5} = 30\text{cm}$$

**Consider the action of a convex lens B**

The image formed by lens **A** acts as a virtual object for lens **B**.

Thus u = -(30 – 20)cm = -10cm and f = 15cm

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ gives

$$v = \frac{f \cdot u}{u - f} = \frac{15 \times -10}{-10 - 15} = 6\text{cm}$$

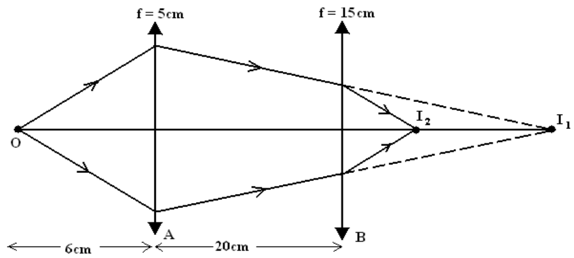
$\Rightarrow$ The image is real and is **6cm** from **B**

$$\text{Required magnification } = m_1 \times m_2$$

$$= \frac{v_1}{u_1} \times \frac{v_2}{u_2}$$

$$= \frac{30}{6} \times \frac{6}{10}$$

$$= \mathbf{3}$$




**5.** A thin converging lens **P** of focal length **10cm** and a thin diverging lens **Q** of focal length **15cm** are placed coaxially **50cm** apart. If an object is placed **12cm** from **P** on the side remote from **Q.**

**(i)** Find the position, nature and magnification of the final image.

**(ii)** Sketch a ray diagram to show the formation of the final image.

**Consider the action of a converging lens P.**

u = 12cm and f = 10cm

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ gives

$$v = \frac{f \cdot u}{u - f} = \frac{12 \times 10}{12 - 10} = 60\text{cm}$$

**Consider the action of a diverging lens Q**.

The image formed by lens **P** acts as a virtual object for lens **Q.**

Thus u = $^-$(60 - 50) = -10cm and f = -15cm

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ gives

$$v = \frac{f \cdot u}{u - f} = \frac{-15 \times -10}{-10 - {}^-15} = 30\text{cm}$$

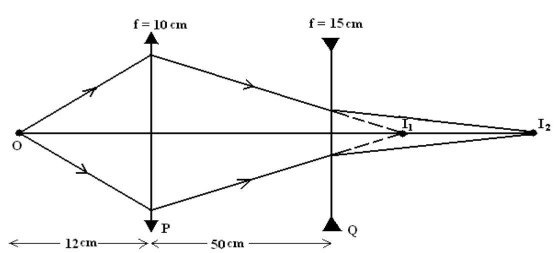
$\Rightarrow$ The image is real and is **30cm** from **Q.**

$$\text{Required magnification } = m_1 \times m_2$$

$$= \frac{v_1}{u_1} \times \frac{v_2}{u_2}$$

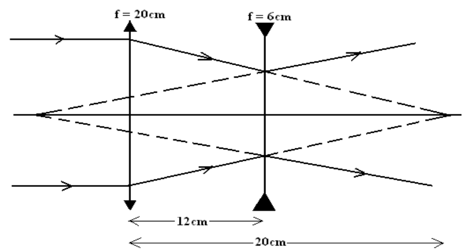
$$= \frac{60}{12} \times \frac{30}{10}$$

$$= \mathbf{15}$$




**6**. Light from a distant object is incident on a converging lens of focal length **20cm**

placed **12cm** in front of a diverging lens of focal length **6cm.** Determine the position and nature of the final image.




**Consider the action of a converging lens**
The image of a distant object is formed at the principal focus of the converging lens

**Consider the action of a diverging lens**
The image formed by a converging lens acts as a virtual object for a diverging lens
$\Rightarrow$ u = -(20 – 12) = -8cm, and f = -6cm

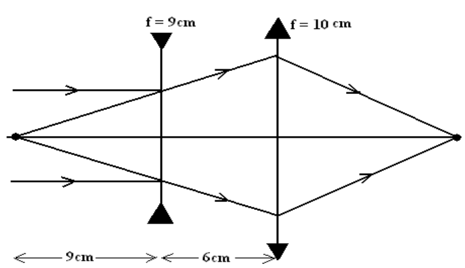
Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ gives

$\Rightarrow$

$$\mathbf{v} = \frac{f \cdot u}{u - f} = \frac{-6 \times -8}{-8 - {}^{-}6} = \mathbf{-24cm}$$

The image is virtual and is 24cm behind the diverging lens.

**7.**. Light from a distant object is incident on a diverging lens of focal length **9cm** placed **6cm** in front of a converging lens of focal length **10cm.** Determine the position and nature of the final image.




**Consider the action of a diverging lens**
The image of a distant object is formed at the principal focus of the diverging lens

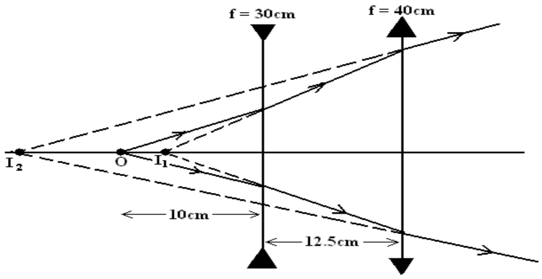
**Consider the action of a converging lens**
The image formed by a diverging lens acts as a real object for a converging lens
$\Rightarrow$ u = (9 + 6)cm = 15cm and f = 10cm

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ gives

$$\mathbf{v} = \frac{f \cdot u}{u - f} = \frac{10 \times 15}{15 - 10} = \mathbf{30cm}$$

⇒ The image is real and is 30cm from the converging lens.

**8**. A thin diverging lens of focal length **30cm** and a thin converging lens of focal length **40cm** are placed coaxially**12·5cm** apart. If an object is placed **10cm** from a diverging lens on the side remote from a converging lens**.** Find the position, nature and magnification of the final image.




**Consider the action of a diverging lens**

u = 10cm, and f = - 30cm

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ gives

$$\mathbf{v} = \frac{f \cdot u}{u - f} = \frac{-30\times 10}{10 - {}^{-}30} = \mathbf{-7\cdot 5cm}$$

**Consider the action of a converging lens**

The image formed by a diverging lens acts as a real object for a converging lens

⇒ u = (75 + 125)cm = 20cm and f = 40cm

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ gives

$$\mathbf{v} = \frac{f \cdot u}{u - f} = \frac{40\times 20}{20 - 40} = \mathbf{-40cm}$$

⇒ The image is virtual and is 40cm from a converging lens.

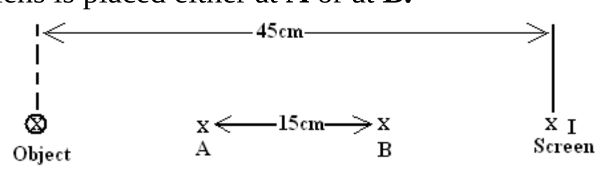
Required magnification $m = m_1 \times m_2$

$$= \frac{v_1}{u_1} \times \frac{v_2}{u_2}$$

$$= \frac{7\cdot 5}{10} \times \frac{40}{20}$$

$$= \mathbf{1\cdot 5}$$

**9**..In the diagram below, the image of the object is formed on the screen when a convex lens is placed either at **A** or at **B.**




If **A** and **B** are **15cm** apart, find the

(i) focal length of the lens.
(ii) magnification of the image when the lens is at **B.**

**Solution**

(i) Using the relation $f = \dfrac{L^2 - d^2}{4L}$ where $L = 45cm$ and $d = 15cm$

$\Rightarrow$ $f = \dfrac{45^2 - 15^2}{4 \times 45}$

$\therefore$ $\mathbf{f = 10cm}$

**(ii)** The magnification $\mathbf{m_2}$ produced by the lens in position **B**

$$\mathbf{m_2} = \frac{L - d}{L + d} = \frac{45 - 15}{45 + 15} = \mathbf{0 \cdot 5}$$

**EXERCISE**

**1**. (i) Define the terms centres of curvature, radii of curvature, principal focus and focal length of a converging lens.

(ii) What are **Sign conventions?**

**2 (a)** An object is placed a distance **u** from a convex lens. The lens forms an image of the object at a distance **v.** Draw a ray diagram to show the path of light when the image formed is:

**(i) real**

**(ii) virtual**

**(b)** Draw a ray diagram to show the formation of an image by a diverging lens.

**(c)** Draw a ray diagram to show the formation of a real image of a virtual point object by a diverging lens

**3**. Give two instances in each case where concave lenses and convex lenses are useful.

**4.** Derive an expression for the focal length **f**, of a convex lens in terms of the object distance **u** and the image distance **v**.

**5**. Define the term **power of a lens.**

**6.** (i) Define the term **linear magnification.**

(ii) Show that the linear magnification produced by a convex lens is equal to the ratio of the image distance to the object distance.

(iii) A convex lens of focal length 15cm forms an erect image that is three times the size of the object. Determine the object and its corresponding image position.

(iv) A convex lens of focal length 10cm forms an image five times the height of its object. Find the possible object and corresponding image positions.

**[Ans: (iii) u = 10cm, v = –30cm (iv) u = 12cm, v = 60cm OR u = 8cm, v =–40cm ]**

**7.** A convex lens forms on a screen a real image which is twice the size of the object. The object and screen are then moved until the image is five times the size of the object. If the shift of the screen is 20cm, determine the
**(i)** focal length of the lens
**(ii)** shift of the object

**[Answers: (i) f = 10cm (ii) 3cm ]**

**8.** A thin converging lens **A** of focal length **6cm** and a thin diverging lens **B** of focal length **15cm** are placed coaxially **14cm** apart. If an object is placed **8cm** from **A** on the side remote from **B,**
**(i)** Find the position, nature and magnification of the final image.
**(ii)** Sketch a ray diagram to show the formation of the final image.

**[ Answers: (i) 30cm from lens B, real image and magnification = 9 ]**

**9.** An object is placed **24cm** in front of a convex lens **P** of focal length **6cm.** When a concave lens **Q** of focal length **12cm** is placed beyond lens **P,** the screen has to be **10cm** away from lens **P** so as to locate the real image formed**.**
**(i)** Find the distance between the two lenses **P** and **Q.**
**(ii)** Sketch a ray diagram to show the formation of the final image**.**

**[ Answer: (i) 4cm ]**

**10.** A lens $\mathbf{L_1}$ casts a real image of a distant object on a screen placed at a distance **15cm** away**.** When another lens $\mathbf{L_2}$ is placed **5cm** beyond lens $\mathbf{L_1}$, the screen has to be shifted by **10cm** further away to locate the real image formed**.** Find the focal length and the type of lens $\mathbf{L_2}$.

**[ Answer: (i) f = –20 cm and therefore the lens is concave ]**

**11.** A thin convex lens is placed between an object and a screen that are kept fixed **64cm** distant apart**.** When the position of the lens is adjusted, a clear focused image is obtained on the screen for two lens positions that are **16cm** distant apart**.**
**(i)** Draw a ray diagram to show the formation of the images in the two lens positions**.**
**(ii)** Find the focal length of the lens
**(iii)** Find the magnification produced in each lens position**.**

**[ Answers: (ii) f = 15cm (iii) $\mathbf{m_1 = 1{\cdot}67}$ , $\mathbf{m_2 = 0{\cdot}6}$ ]**

**12**. (i) State two conditions necessary for a biconvex lens to form a real image of an object.

(ii) Show that the minimum distance between an object and a screen for a real image to be formed on it is **4f,** where **f** is focal length of a convex lens**.** Hence show also that the object and its image are then equidistant from the lens.

**13. (i)** Explain with the aid of a convex lens the term **conjugate foci.**

**(ii)** What is meant by **reversibility of light** as applied to formation of a real image by a convex lens?

**14**. A converging lens of focal length **f** is placed between an object and a screen. The position of the screen is adjusted until a clear magnified image is obtained on the screen. Keeping the screen fixed in this position at a distance **L** from the object, the lens is displaced through a distance **d** to obtain a clear diminished image on the screen.

**(i)** Draw a ray diagram to show the formation of the images in the two cases.

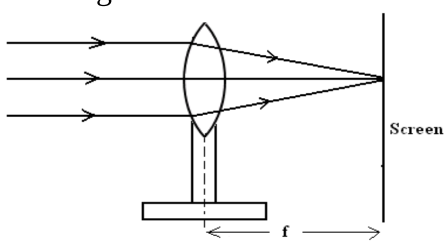
**(ii)** Show that $\mathbf{L^2 - d^2 = 4Lf}$.

**(iii)** Find the product of the magnifications produced in the two cases.

**MEASUREMENT OF FOCAL LENGTH OF A CONVEX LENS**

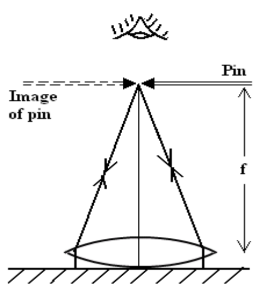
**Method (1) using a distant object**

A distance object such as a window of a tree is focused on to the screen using a convex lens whose focal length is to be determined.



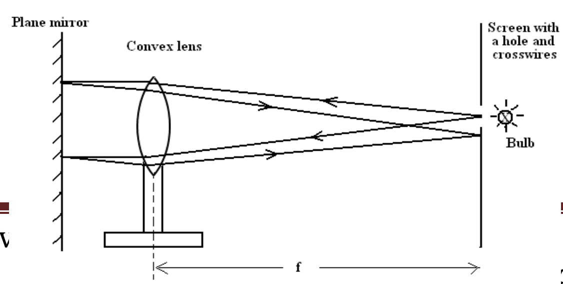

The distance of the screen from the lens is then the focal length **f** of the lens, which can thus be measured

**Method (2) using a plane mirror and the non parallax method**.



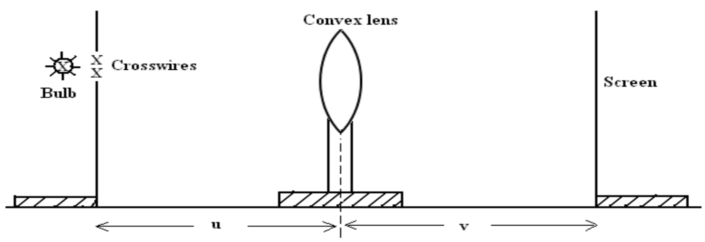

The lens is placed directly on a plane mirror. A pin is clamped horizontally on a retort stand with the apex along the axis of the lens. Move the pin up or down to locate the position where the pin coincides with its image using the method of no parallax. The distance of the pin to the lens is measured and it is equal to the focal length of the lens.

**Method (3) using a plane mirror and an illuminated object**



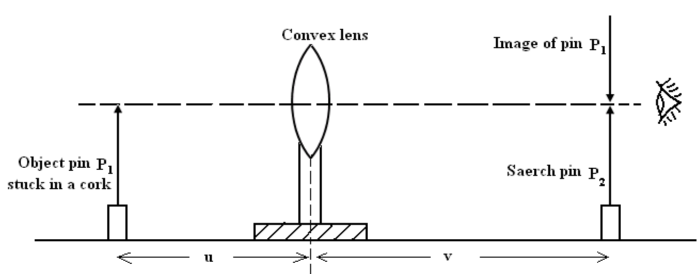
A lens mounted in a holder is placed between a screen with cross wires and a plane mirror as shown above. The lens is moved in between, until a sharp image of the cross-wire is formed on the screen besides the object. The distance **f** of the lens from the screen is then the focal length of the lens, which can thus be measured.

**<u>Method (4) using the thin lens formula  and an illuminated object.</u>**



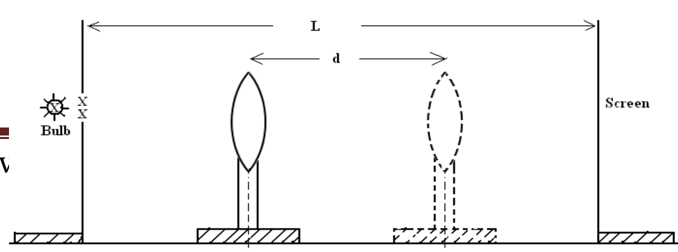

An illuminated object **O** is placed at a distance **u** in front of a mounted convex lens.
The position of the screen is adjusted until a sharp image of **O** is formed on the screen at a distance **v** from the lens.   The procedure is repeated for several values of u and the results are tabulated including values of **uv** and **u + v**. A graph of **uv** against **u + v** is plotted and the slope **s** of such a graph is equal to the focal length **f**
of the lens.

**<u>Method (5) using the thin lens formula  and the method of no-parallax.</u>**




An object pin $P_1$ is placed at a distance **u** in front of a mounted convex lens so that its tip lies along the axis of the mirror. A search pin $P_2$ placed behind the lens is adjusted until it coincides with the image of pin $p_1$ by no-parallax method. The distance **v** of pin $p_2$ from the lens is measured**.** The procedure is repeated for several values of **u** and the results are tabulated including values of **uv**, and **u+v.** A graph of **uv** against
**u+v** is plotted and the slope **s** of such a graph is equal to the focal length **f** of the lens.

**<u>Method (6) using displacement method</u>**

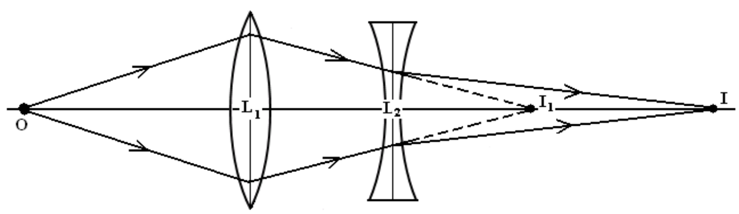


An illuminated object **O** is placed in front of a convex lens in position **A.** The position of the screen is adjusted until a sharp magnified image is formed on the screen. Keeping the screen and the object fixed in this position at a distance **L** apart, the lens is displaced through a measured distance **d** to position **B** where another sharp diminished image is formed on the screen. The focal length **f** of the lens can then be calculated from $\mathbf{f} = \frac{\mathbf{L^2 - d^2}}{\mathbf{4L}}$. The procedures are repeated for different values of **L** and the average value of **f** is calculated.

**NOTE:**
Since no measurements need be made to the surfaces of the lens in Method (5), then it is most useful when finding the focal length of: **(i)** a thick lens
**(ii)** an inaccessible lens, such as that fixed inside an eye-piece or telescope tube.

**MEASUREMENT OF FOCAL LENGTH OF A DIVERGING LENS**

**Method (1) using a converging lens**



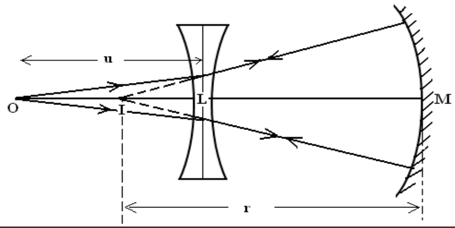

An object **O** is placed in front of a converging lens of known focal length so as to obtain a real image on the screen at $\mathbf{I_1}$. The distance $\mathbf{L_1I_1}$ of the screen from the converging lens is measured. A diverging lens whose focal length is required is placed between the screen and the converging lens. The screen is moved to obtain a new real image **I** on to it. The distances $\mathbf{L_1L_2}$ and $\mathbf{L_2I}$ are measured. The focal length of the diverging lens can then be calculated from

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ where $v = L_2I$ and

$u = -(L_1I_1 - L_1L_2)$ .

**NOTE:**
When a diverging lens is placed in between $\mathbf{L_1}$ and $\mathbf{L_2}$ the image $\mathbf{I_1}$ formed by a converging lens acts as a virtual object for the diverging lens hence giving the final real image **I.**

**Method (2) using a concave mirror method**



A diverging lens whose focal length is required is placed coaxially with a concave mirror of known radius of curvature **r**. An object **O** is placed in front of the lens. The position of the object is adjusted until it coincides with its image. The distances **OL** and **LM** are measured. The focal length of the diverging lens can then be calculated from $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ where **u = OL** and **v = - (r – LM).**

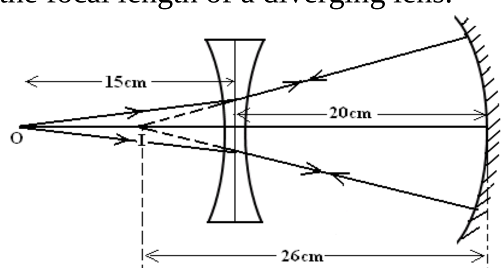
**NOTE:**
As the object and its image are coincident at **O,** the rays must be incident normally on the mirror and there fore retrace their own path through the centre of curvature of the mirror at **I** and this is the position of the virtual image.

**EXAMPLES:**

**1**. An object is placed **15cm** in front of a diverging lens placed coaxially with a concave mirror of focal length **13cm.** When the concave mirror is **20cm** from the lens the final image coincides with the object.

**(i)** Draw a ray diagram to show how the final image is formed.

**(ii)** Determine the focal length of a diverging lens.



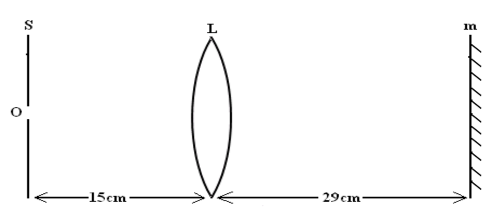

Consider the action of a diverging lens

**u = 15cm** and **v = -(26 - 20)cm = -6cm '**virtual image'.

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ gives

$$\mathbf{f} = \frac{uv}{u+v} = \frac{15\times -6}{15+-6} = \mathbf{-10cm}$$

**2**.A convex lens **L**, a plane mirror **M** and a screen **S** are arranged as shown below so that a sharp image of an illuminated object **O** is formed on the screen **S**.

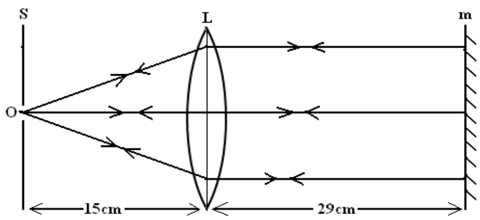


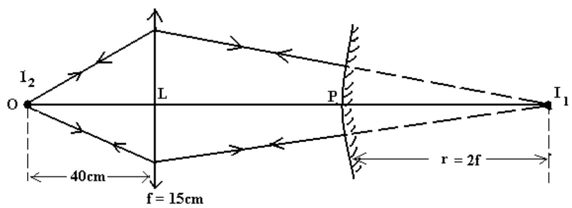

When the plane mirror is replaced by a convex mirror, the lens has to be moved **25cm** towards the mirror so as to obtain a sharp focused image on the screen.

**(i)** Illustrate the two situations by sketch ray diagrams.

**(ii)** Calculate the focal length of the convex mirror.

**Solution**




Consider the action of a convex lens

u = 40cm, and f = 15cm

Using the lens formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ gives

$$v = \frac{f \cdot u}{u - f} = \frac{15 \times 40}{40 - 15} = \mathbf{24cm}$$

The radius of curvature **r = (24 – 4)cm = 20cm**

**Using the relation r = 2f**

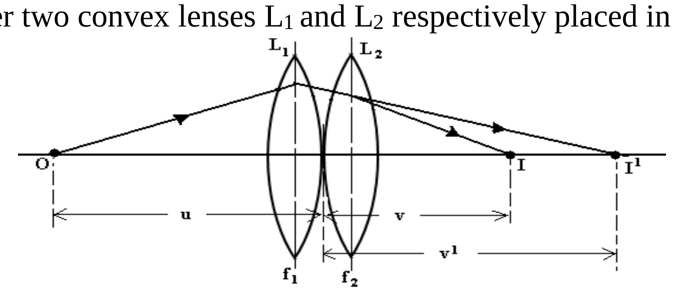
$\Rightarrow$ 2f = 20cm

$\therefore$ f = 10cm

**Thus f = -10cm** "The centre of curvature of a convex mirror is virtual"

**COMBINED FOCAL LENGTH OF TWO LENSES IN CONTACT.**

Consider two convex lenses $L_1$ and $L_2$ respectively placed in contact as shown:




In the absence of lens$L_2$, lens $L_1$ forms a real image at $I^1$

Using the lens formula for $L_1$

$$\frac{1}{f_1} = \frac{1}{u} + \frac{1}{v^1} \quad \text{--------------------(i)}$$

When lens $L_2$ is now laced in contact with lens $L_1$, the image $I^1$ acts as a virtual object for lens $L_2$ to give a final real image I.

Using the lens formula for $L_2$

$$\frac{1}{f_2} = \frac{1}{-v^1} + \frac{1}{v} \text{ -------------------(ii)}$$

Adding (i) and(ii) gives.

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{u} + \frac{1}{v^1} - \frac{1}{v^1} + \frac{1}{v} \qquad \Rightarrow$$

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{u} + \frac{1}{v}$$

But $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

**Thus** $\mathbf{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}$ where **f** is focal length of the combined lenses.

NOTE:

This formula for f applies for any two lenses in contact such as two diverging lenses or a converging and diverging lens.

When the formula is used the signs of the focal length must be considered as illustrated below.

Suppose a converging lens of focal length 8cm is placed in contact with a diverging lens of focal length 12cm then $F_1 = {}^{+}8$cm and $f_{2=} = {}^{-}12$cm

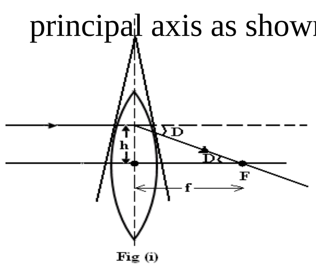
Using the relation $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ gives

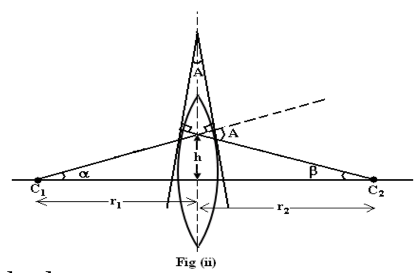
$$\mathbf{f} = \frac{f_1 \cdot f_2}{f_1 + f_2} = \frac{8 \times -12}{8 + {}^{-}12} = \mathbf{24cm}$$

The positive sign shows that the combination acts as a convex lens.

**FULL FORMULA FOR A THIN LENS**

Consider in each case a paraxial ray incident on the same lens at a small height **h** above the principal axis as shown:




**C1** and **C2** are the centers of curvature of the lens.

From Fig (i), the ray is converged to the focal point **F** to suffer a small deviation **D**

where $\mathbf{D} \approx \tan D = \mathbf{\frac{h}{f}}$ ---------------------------------- (i)

From Fig (ii), the radii normal to the lens surface at **P** and **Q** pass through the centres of curvature **C1** and **C2**. From geometry, $\mathbf{A = \alpha + \beta}$ where $\boldsymbol{\alpha} \approx \tan \alpha = \mathbf{\frac{h}{r_1}}$ and $\boldsymbol{\beta} = \tan \beta = \mathbf{\frac{h}{r_2}}$

$$\Rightarrow \qquad A = \frac{h}{r_1} + \frac{h}{r_2} \quad \text{--------------------(ii)}$$

From the theory of prisms, all rays incident on a lens of refractive index **n** suffer the same small deviation **D = (n - 1) A** ----------------------------(iii)

Combining equation (i) (ii) and (iii) gives

$$\frac{h}{f} = (n-1)\left(\frac{h}{r_1} + \frac{h}{r_2}\right)$$

**Thus** $$\mathbf{\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)}$$

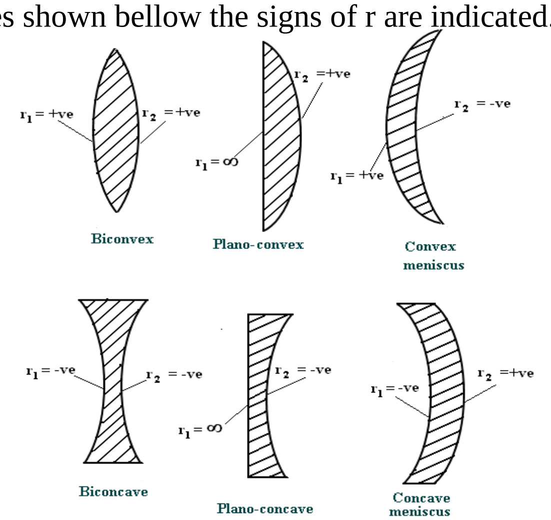
NOTE:

A more general form of the formula is given by $\mathbf{\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)}$

Where $\mathbf{n_2}$ is the refractive index of the lens material and $\mathbf{n_1}$ is the refractive index of surrounding medium.

## SIGN CONVENTION

If the surface of the lens is convex, then the corresponding radii of curvature is positive, however if the surface is concave then its corresponding radius of curvature is negative. Thus for the type of lenses shown bellow the signs of r are indicated.




NOTE:

In numerical work for a convex meniscus, the radius of curvature with the largest magnitude takes up the negative sign.

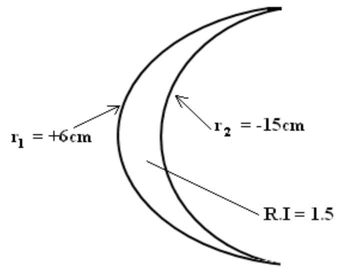
EXAMPLE:

**1.** A converging meniscus with radii of curvature **15cm** and **6cm** is made of glass of refractive index **1·5.** Calculate its focal length when surrounded by:

**(i)** air

**(ii)** a liquid of refractive index **1·2**

**Solution:**
**(i)**




$r_1$ =6cm, $r_2$ = -15cm and n =1·5

Using the relation $\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$ gives

$$\frac{1}{f} = (1\cdot 5 - 1)\left(\frac{1}{6} - \frac{1}{15}\right)$$

$\Rightarrow$ $\mathbf{f = 20cm}$

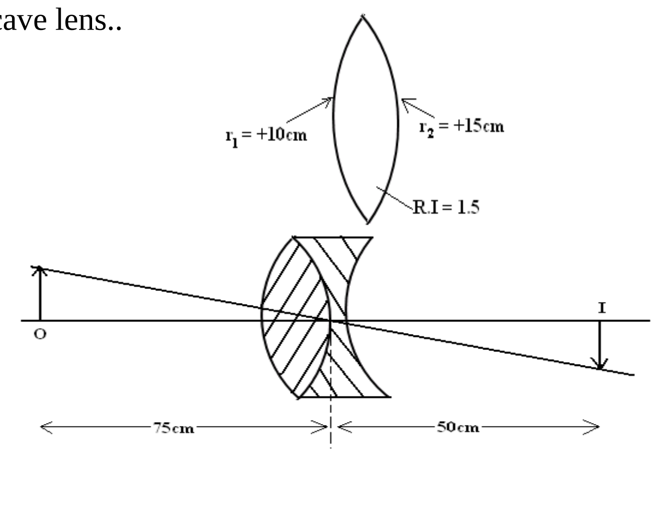
**(ii)** Using the relation $\frac{1}{f} = \left(\frac{n_g}{n_L} - 1\right)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$ gives

$$\frac{1}{f} = \left(\frac{1\cdot 5}{1\cdot 2} - 1\right)\left(\frac{1}{6} - \frac{1}{15}\right)$$

$\Rightarrow$ $\mathbf{f = 40cm}$

.

**2.**A thin concave lens is placed in contact with a convex lens made of glass of refractive index **1·5** and its surfaces have radii of curvature **10cm** and **15cm**. If an object placed **75cm** in front of the lens combination gives rise to an image on a screen at a distance **50cm** from the combination, calculate the focal length of the concave lens..



**Using the lens formula** $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ **gives**

$$f = \frac{uv}{u+v} = \frac{75\times 50}{75+50} = 30cm$$

$\therefore$ **For the lens combination, f = 30cm**

**For the convex lens** $\frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

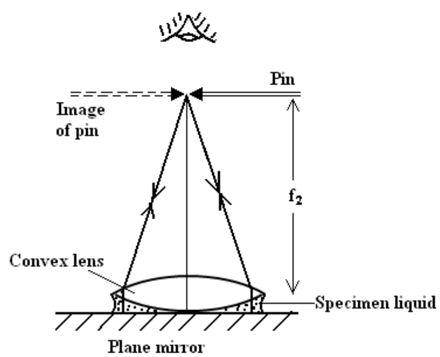
$$\therefore \quad \frac{1}{f_1} = (1\cdot 5 - 1)\left(\frac{1}{10} + \frac{1}{15}\right)$$

$$\Rightarrow \quad f_1 = 12cm$$

But $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

$\Rightarrow$ **focal length of a concave lens** $f_2 = \frac{f_1 f}{f_1 - f} = \frac{12\times 30}{12-30} = -20cm$

**DETERMINATION OF THE REFRACTIVE INDEX OF A LIQUID USING A CONVEX LENS OF KNOWN RADII OF CURVATURE**




The focal length of the lens is first determined by placing it directly on a plane mirror. Clamp a pin horizontally on a retort stand with its apex along the axis of the lens. Move the pin up or down to locate the position where the pin coincides with its image using the method of the no parallax. Measure the distance of $f_1$ of the pin from the lens.

Remove the lens and place a little quantity of the specimen liquid on the plane mirror. Replace the convex lens and then locate the new position where the pin coincides with the image. Measure the distance $f_2$ of the pin from the lens. If $f_L$ is the focal length of the liquid then

$\frac{1}{f_2} = \frac{1}{f_1} + \frac{1}{f_L}$ and $\frac{1}{f_L} = (n_L - 1)\left(\frac{1}{-r}\right)$ hence the refractive index of the liquid can be calculated from $n_L = 1 - \frac{r}{f_L}$ where r is the radius of curvature of the convex lens

NOTE:

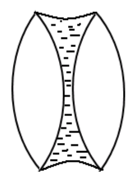
It can be seen from the experiment that the liquid lens is Plano concave type with its lower surface corresponding to the plane surface and the upper surface to the convex lens. There fore

$r_1$ = -ve r is the radius of curvature of the upper surface and $r_2 = \alpha$ is the radius of curvature of the lower surface.

**EXAMPLES:**

**1.** Two equiconvex lenses of focal length **20cm** and made of glass of refractive index **1·6** are placed in contact and the space between them is filled with a liquid of refractive index **1·4**. Find the focal length of the lens combination.

**Solution:**



**Using the relation** $\frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

For an equiconvex lens, $r_1 = r_2 = r$

$$\Rightarrow \quad \frac{1}{20} = (1\cdot6 - 1)\left(\frac{2}{r}\right)$$

$$\therefore \quad \mathbf{r = 24cm}$$

**For the equiconcave water lens,** $\frac{1}{f_w} = (n_w - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

$r_1 = r_2 = -24\text{cm}$

$$\Rightarrow \quad \frac{1}{f_w} = (1\cdot4 - 1)\left(\frac{1}{-24} + \frac{1}{-24}\right)$$

$$\therefore \quad \mathbf{f_w = -30cm}$$

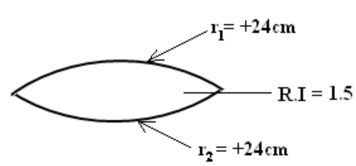
For the lens combination, $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_w}$

But $f_1 = f_2 = 20\text{cm}$

$$\therefore \quad \frac{1}{f} = \frac{1}{20} + \frac{1}{20} + \frac{1}{-30}$$

**Thus focal length of the lens combination** $\mathbf{f = 15cm}$

**2.** A thin equi-convex lens of glass of refractive index = 1.5 whose surface have radii of curvature 24cm is laced on a horizontal plane mirror when the space between the mirror and the lens is filled with a liquid, a pin placed 40cm vertically above the lens coincides with its image. Calculate the refractive index of the liquid.



Using the relation $\frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$ gives

$$\frac{1}{f_1} = (1{\cdot}5 - 1)\left(\frac{1}{24} + \frac{1}{24}\right)$$

$\Rightarrow$ $\mathbf{f_1 = 24cm}$

Consider the liquid-lens combination

Using the relation $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_L}$ gives

$$\frac{1}{40} = \frac{1}{24} + \frac{1}{f_L}$$

$\Rightarrow$ $\mathbf{f_L = -60cm}$

**Using the relation** $\mathbf{\frac{1}{f_L}} = (n_L - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

$$\frac{1}{-60} = (n - 1)\left(\frac{1}{-24} + \frac{1}{\infty}\right)$$

**Hence** $\mathbf{n_L = 1{\cdot}4}$

**3.** The curved face of a Plano convex lens of refractive index 1.5 is placed in contact with a plane mirror. A pin placed at a distance 20cm coincides with its image.
A film of a liquid is now introduced between the lens and the plane mirror. Then the coincidence of the pin and its image is found to be at a distance 100cm. Calculate the refractive index of the liquid.

Solution:

Using the relation $\frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$ gives

$$\frac{1}{20} = (1{\cdot}5 - 1)\left(\frac{1}{\infty} + \frac{1}{r_2}\right)$$

$\Rightarrow$ $r_2 = \mathbf{10cm}$

Consider the **liquid-lens** combination.

Using the relation $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_L}$ gives

$$\frac{1}{100} = \frac{1}{20} + \frac{1}{f_L}$$

$\Rightarrow$ $\mathbf{f_L = -25cm}$

Consider the liquid lens.

Using the relation $\frac{1}{f_L} = (n_L - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

$$\Rightarrow \quad \frac{1}{-25} = (n_L - 1)\left(\frac{1}{\infty} + \frac{1}{-10}\right)$$

$\therefore$ $\mathbf{n_L = 1 \cdot 4}$

**4**.A small quantity of a liquid of refractive index 1.4 is poured on a horizontal plane mirror and a biconvex lens of focal length 30cm and refractive index 1.5 is then placed on top of the liquid. The pin is moved along the axis of the lens until no parallax between it and its image find the distance between the pin and the lens.

Using the relation $\frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

But for a biconvex lens $r_1 = r_2 = r$

$$\Rightarrow \quad \frac{1}{f_1} = (n_L - 1)\left(\frac{2}{r}\right)$$

$$\frac{1}{30} = (1 \cdot 5 - 1)\left(\frac{2}{r}\right)$$

$\therefore$ $\mathbf{r = 30cm}$

For the liquid lens $\frac{1}{f_L} = (n_L - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$ where $n_L = 1 \cdot 4$

$$\Rightarrow \quad \frac{1}{f_L} = (1 \cdot 4 - 1)\left(\frac{1}{-30} + \frac{1}{\infty}\right)$$

$\therefore$ $\mathbf{f_L = -75cm}$

For the liquid - lens combination $\frac{1}{f} = \frac{1}{30} + \frac{1}{-75}$

$\Rightarrow$ $\mathbf{f = 50cm}$

The distance between the pin and the lens is **50cm.**

**5.** In an experiment to determine the refractive index of paraffin the apparatus was first set up as shown using a convex lens of focal length **f:**

Some water of refractive index $\mathbf{4/3}$ was placed on the mirror and the lens on top. A pin placed at a height $\mathbf{h_1}$ vertically above the lens coincides with its image. The experiment was repeated using paraffin instead of water and the new position of coincidence was found to be at a height $\mathbf{h_2}$. Show that the refractive index $\mathbf{n_p}$ of paraffin is given by $\mathbf{n_p = 1 + \dfrac{h_1(h_2 - f)}{3h_2(h_1 - f)}}$.

For the water - lens combination, $\dfrac{1}{h_1} = \dfrac{1}{f_w} + \dfrac{1}{f}$ -------------(i)

But $\dfrac{1}{f_w} = (n_w - 1)\left(\dfrac{1}{-r_1} + \dfrac{1}{\infty}\right)$ where $n = 4/3$

$$\Rightarrow \quad \frac{1}{f_w} = \left(4/3 - 1\right)\frac{1}{-r_1}$$

$$\therefore \quad \frac{1}{f_w} = \frac{-1}{3r_1}$$

Equation (i) now becomes $\dfrac{1}{h_1} = \dfrac{-1}{3r_1} + \dfrac{1}{f}$

$$\Rightarrow \quad r_1 = \frac{h_1 f}{3(h_1 - f)} \quad \text{-------------------------(ii)}$$

For the paraffin - lens combination, $\dfrac{1}{h_2} = \dfrac{1}{f_p} + \dfrac{1}{f}$ -----------------(iii)

But $\dfrac{1}{f_p} = (n_p - 1)\left(\dfrac{1}{-r_1} + \dfrac{1}{\infty}\right)$

$$\Rightarrow \quad \frac{1}{f_p} = \frac{1 - n_p}{r_1}$$

Equation (iii) now becomes $\dfrac{1}{h_2} = \dfrac{1 - n_p}{r_1} + \dfrac{1}{f}$

$$\Rightarrow \quad n_p = 1 + \left(\frac{h_2 - f}{h_2 f}\right) r_1 \quad \text{--------------------(iv)}$$

Substituting equation (ii) in (iv) gives

$$n_p = 1 + \left(\frac{h_2 - f}{h_2 f}\right)\left(\frac{h_1 f}{3(h_1 - f)}\right)$$

**Thus** $\quad \mathbf{n_p = 1 + \dfrac{h_1(h_2 - f)}{3h_2(h_1 - f)}}$

.

**DEFECTS IN IMAGES (ABERRATIONS)**

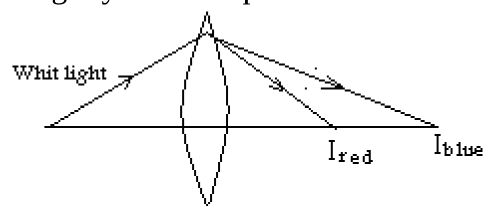
This is the distortion of images formed by either spherical mirrors or spherical lenses.
When mirrors and lenses under consideration are of large aperture, images formed by them can differ in shape and color from the object. Such defects are known as aberration or defects in images. There are two types of aberrations namely:

**(i)** Chromatic aberration.

**(ii)** Spherical aberration.

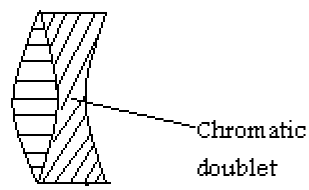
**CHROMATIC ABERRATION**

This is when a beam of white light incident on a convex lens after being dispersed produces coloured images of an object at slightly different points on lens axis.




**ACHROMATIC COMBINATION OF LENSES**

Chromatic aberration can be reduced by using an achromatic doublet. This consists of a convex lens combined with a concave lens made from different glass materials. The convex lens deviates the rays while the concave lens nullifies the diversion.




**NOTE:**

The distance $\mathbf{F_r\,F_v}$ **(i.e.** $\mathbf{f_r - f_v}$**)** is the longitudinal chromatic aberration for the lens.

**SPHERICAL ABERRATION**

This is when a wide beam of light incident on **either** a lens **or** a mirror of large aperture produces a distorted image due to rays being convergent to different points on the principal axis. This is as a result of the marginal rays being converged nearer the lens or mirror than the paraxial rays.

In lenses, Spherical aberration can be reduced by using a circular stop to cut off marginal rays.

**NOTE:**

A circular stop is an opaque disc having a hole in the middle for allowing in only paraxial rays incident on the lens

In mirrors, spherical aberration can be minimized by using a paraboloidal mirror.
This because a paraboloidal mirror converges a wide parallel beam of light incident onto its surface to a single focus as shown.

**COMPARISION OF NARROW AND WIDE APERTURE LENSES**

Lenses of narrow aperture are widely used in optical instruments so as to avoid spherical aberration. This is because when a wide beam of light falls on a lens of narrow aperture, all rays are paraxial and are thus brought to a single focus to form a sharp image. However a lens with a wide aperture allows in both paraxial and marginal rays, which are thus brought to different focus to form a blurred image.

**EXAMPLE:**

**1**.The curved surface of a plane convex lens has a radius of curvature of **20cm** and is made of crown glass for which the refractive index of red and blue light respectively is **1·5** and **1·52** calculate the longitudinal chromatic aberration for the lens.

**Using the relation** $\frac{1}{f} = (n - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

For red light, $\frac{1}{f_r} = (1\cdot5 - 1)\left(\frac{1}{20} + \frac{1}{\infty}\right)$

$\Rightarrow \quad \mathbf{f_r = 40cm}$

For blue light, $\frac{1}{f_b} = (1\cdot52 - 1)\left(\frac{1}{20} + \frac{1}{\infty}\right)$

$\Rightarrow \quad \mathbf{f_b = 38\cdot5cm}$

**Thus longitudinal aberration = fr – fb**

**= (40 - 38·5 )cm**

**= 1·5cm**

**2**.A convex lens of radius of curvature **24cm** is made of glass of refractive index for red and violet light of **1·6** and **1·8** respectively. A small object illuminated with white light is placed on the axis of the lens at a distance **45cm** from the lens. Find the separation of the images formed in the red and violet constituents of light.

**Using the relation** $\frac{1}{f} = (n - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

For an equiconvex lens, $r_1 = r_2 = r$

$\Rightarrow$ For red light, $\frac{1}{f_r} = (1\cdot8 - 1)\left(\frac{1}{24} + \frac{1}{24}\right)$

$\therefore \quad \mathbf{f_r = 30cm}$

For violet light, $\frac{1}{f_v} = (1\cdot6 - 1)\left(\frac{1}{24} + \frac{1}{24}\right)$

$\therefore \quad \mathbf{f_v = 20cm}$

**Using the lens formula** $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

**For red light** $\quad v_r = \frac{f_r u}{u - f_r} = \frac{30 \times 45}{45 - 30} = 90cm$

**For violet light** $\quad v_v = \dfrac{f_v u}{u - f_v} = \dfrac{20 \times 45}{45 - 20} = 36\text{cm}$

$\therefore$ **Image separation** $= v_r - v_v$

$= (90 - 36)\text{cm}$

$= 54\text{cm}$

**EXERCISE:**

**1**.Describe how the focal length of a convex lens can be determined using a plane mirror and the non-parallax method.

**2**.You are provided with the following pieces of apparatus: A screen with cross wires, a lamp, a convex lens, a plane mirror, and a meter ruler. Describe an experiment to determine the focal length of a convex lens using the above apparatus.

**3**. Describe an experiment, including a graphical analysis of the results to determine the focal length of a convex lens using a no parallax method.

**4.** Describe an experiment to determine the focal length of a thick convex lens having inaccessible surfaces.

**5**.A convex lens is contained in a cylindrical tube such that its exact position in the tube is not accessible. Describe how you would determine the focal length of the lens with out removing it from the tube

**6**.Describe how the focal length of a diverging lens can be determined using a convex lens.

**7**.Describe how the focal length of a concave lens can be obtained using a concave mirror.

**8**.Derive an expression for the focal length of a combination of two thin converging lenses in contact, in terms of their focal lengths.

**9. (a)** Show that the focal length **f** of a thin convex lens in air is given by

$\dfrac{1}{f} = (n - 1)\left(\dfrac{1}{r_1} + \dfrac{1}{r_2}\right)$, Where **n** is the refractive index of the material of the lens, $r_1$ and $r_2$ are the radii of curvature of the surfaces of the lens.

**(b)** The radii of curvature of the faces of a thin convex meniscus lens of glass of refractive index **1·75** are **8cm** and **12cm**. Calculate the focal length of the lens when completely surrounded by a liquid of refractive index **1·25.**

**[ Answer: f = 60cm ]**

**10. (a)** Describe, giving the relevant equations, how the refractive index of a liquid can be determined using a convex lens of known radius of curvature.

**(b)** A biconvex lens of radius of curvature **24cm** is placed on a liquid film on a plane mirror. A pin clamped horizontally above the lens coincides with its image at a distance of **40cm** above the lens. If the refractive index of the liquid is **1·4**, calculate the refractive index of the material of the lens.

**[ Answer: n = 1·5 ]**

**11. (a)** Differentiate between **chromatic** and **spherical aberrations.**

**(b)** Explain how the defects in **10(a) above** can be minimized in practice.

**(c)** Explain why lenses of narrow aperture are preferred to lenses of wide aperture in optical instruments.

**(d)** Draw a ray diagram showing the reflection of a wide beam of light by a concave mirror of wide aperture

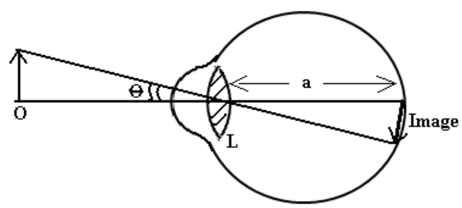
**OPTICAL INSTRUMENTS**

These are instruments which work on the principle of reflection and refraction of light rays. They include telescopes, microscopes, prism binoculars, camera and projection lantern.

**VISUAL ANGLE**

This is the angle subtended to the eye by the object.

Consider an object **O** placed at some distance from the eye and subtending a small angle $\theta$ to the eye as shown.



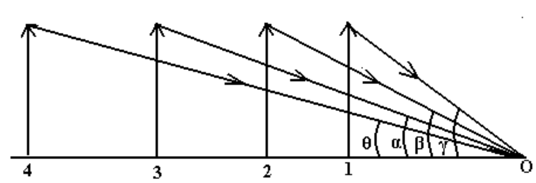

The opposite angles at **L** are equal thus the height **h** of the image on the retina is given by $\mathbf{h = a\theta}$ where **a** is a fixed distance from **L** to the retina

$\Rightarrow \mathbf{h \propto \theta}$

**Thus the height of the image formed by the eye on the retina is proportional to the angle subtended at the eye by the object. (i.e. the greater the visual angle, the greater is the apparent size of the object).**

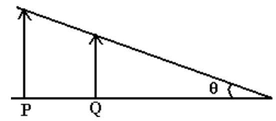
**APPARENT SIZE OF COLLINEAR OBJECTS**

Consider **4** vertical poles of the same height placed at different positions from an observer at **O** as shown:




The poles that are nearer to the observer subtend bigger angles at the observer's eye than the furthest pole **(No 4).** Since the apparent height of the object is proportional to the angle it subtends at the eye, the furthest pole appears shortest.

However if objects say **P** and **Q** subtend the same angle at the observer's eye, the object appear to be of the same size although the furthest object **P** is physically bigger than that at **Q** as shown




**ANGULLAR MAGNIFICATION OR MAGNIFYING POWER OF AN OPTICAL INSTRUMENT**

This is the ratio of the angle subtended at the eye by the final image formed when using the instrument to the angle subtended at the unaided eye by the object.

**NOTE**:

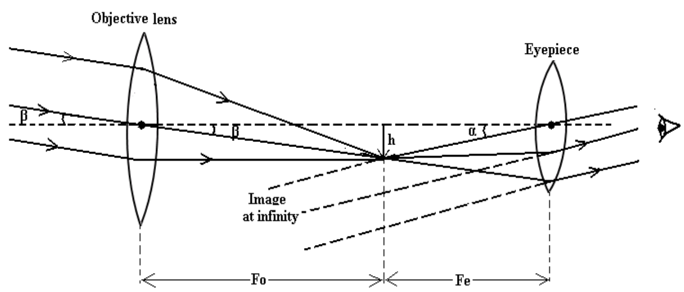
Unaided eye is when the object is viewed without using an instrument.

A telescope is a device used for viewing distant objects.

A telescope is in normal adjustment when the final image of a distant object is formed at infinity.

An astronomical telescope consists of f two converging lenses; one is an objective of long focal length and the other an eyepiece of short focal length. This enables a high angular magnification to be obtained.

In normal adjustment, the objective forms a real inverted image of a distant object at its focal point $\mathbf{F_o}$ situated exactly at the principal focus $\mathbf{F_e}$ of the eyepiece. This intermediate image acts as a real object for the eyepiece to give rise to a final virtual image at infinity as shown.




Let **h** be the height of the intermediate image formed at $\mathbf{F_o}$.

**Magnifying power** $M = \dfrac{\alpha}{\beta}$

$\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are small angles such that $\boldsymbol{\alpha} \approx \tan\alpha = \mathbf{h}/\mathbf{f_e}$ and $\boldsymbol{\beta} \approx \tan\beta = \mathbf{h}/\mathbf{f_o}$

$$\Rightarrow \qquad M = \frac{h/f_e}{h/f_o}$$

$$\therefore \qquad \mathbf{M} = \frac{\mathbf{f_o}}{\mathbf{f_e}}$$

For a high magnifying power, the objective should have a long focal length and the eyepiece a short focal length.

The above expression for the magnifying power is only true for a telescope in normal adjustment with lens separation $\mathbf{f_o} + \mathbf{f_e}$.

**NOTE:**

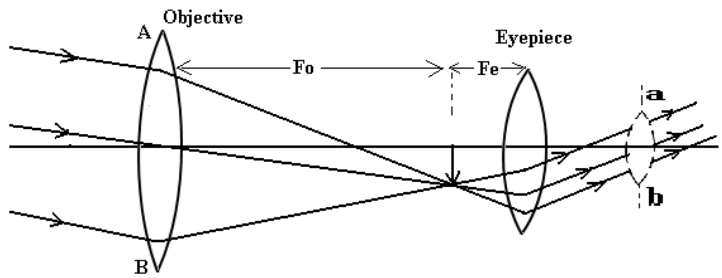
**(i)** The angle $\boldsymbol{\beta}$ subtended at the eye by the distant object is practically equal to the angle subtended at the objective.

**(ii)** From above, the distance between the lenses is equal to the sum $(\mathbf{f_o} + \mathbf{f_e})$ of their focal lengths. This provides a simple method of setting up two convex lenses to form an astronomical telescope when their focal lengths are known.

**THE EYE RING OR EXIT PUPIL AND ITS RELATION TO ANGULAR MAGNIFICATION FOR A TELESCOPE IN NOMAL ADJUSTME**NT.

Eye ring is the best position of the eye when using an optical instrument and it is the image of the objective formed by the eyepiece.

At the exit pupil, the eye receives a maximum amount of light entering the objective from out side so that its field of view is greatest.




At the exit pupil, the eyepiece forms an image of the objective lens.

Consider the action of the eyepiece

$\mathbf{u} = \mathbf{f_o} + \mathbf{f_e}$ and $\mathbf{f} = \mathbf{f_e}$

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ gives

$$\mathbf{v} = \frac{f \cdot u}{u - f} = \frac{f_e(f_o + f_e)}{(f_o + f_e) - f_e} = \frac{\mathbf{f_e(f_o + f_e)}}{\mathbf{f_o}}$$

$$\Rightarrow \quad \frac{\textbf{diameter of objective}}{\textbf{diameter of eye-ring}} = \frac{AB}{A^1B^1} = \frac{u}{v} = \frac{f_o + f_e}{f_e(f_o + f_e)/f_o} = \frac{\mathbf{f_o}}{\mathbf{f_e}}$$

But the magnifying power $\mathbf{M} = \frac{\mathbf{f_o}}{\mathbf{f_e}}$ for a telescope in normal adjustment

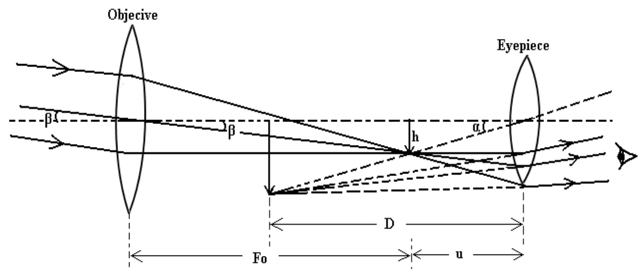
$$\textbf{Hence} \quad \mathbf{M} = \frac{\textbf{diameter of objective}}{\textbf{diameter of eye-ring}} = \frac{\mathbf{f_o}}{\mathbf{f_e}}$$

The above expression for the magnifying power is only true for a telescope in normal adjustment with lens separation $\mathbf{f_o} + \mathbf{f_e}$.

**REFRACTING ASTRONAMICAL TELESCOPE WITH FINAL IMAGE AT NEAR POINT**

When the final image is at near point, then the telescope is not in normal adjustment.

The objective forms a real inverted intermediate image of a distant object at its focal point $\mathbf{F_o}$ and it is a distance **u** from the eye piece nearer than its focal length $f_e$. This intermediate image acts as a real object for the eyepiece to give rise to a final virtual image between the objective and the intermediate image **I** at a distance **D** from the eyepiece. as shown.:




Let **h** be the height of the intermediate mage formed at $\mathbf{F_o}$.

Angular magnification $M = \dfrac{\alpha}{\beta}$

$\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are small angles such that $\boldsymbol{\alpha} \approx \tan\alpha = \mathbf{h/u}$ and $\boldsymbol{\beta} \approx \tan\beta = \mathbf{h/f_o}$

$$\Rightarrow \qquad M = \frac{h/u}{h/f_o}$$

$$\therefore \qquad \mathbf{M} = \frac{\mathbf{f_o}}{\mathbf{u}} \quad \text{--------------------(i)}$$

Consider the action of the eyepiece:

$\mathbf{v} = \mathbf{-D}$ and $\mathbf{f} = \mathbf{f_e}$

Using the lens formula $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$ gives

$$\mathbf{u} = \frac{f \cdot v}{v - f} = \frac{f_e \times -D}{-D - f_e} = \frac{\mathbf{f_e D}}{\mathbf{D + f_e}}$$

Equation (i) now becomes

$$\mathbf{M} = \frac{f_o}{f_e D / (D + f_e)} = \frac{f_o(D + f_e)}{f_e D}$$

$$\Rightarrow \qquad \mathbf{M} = \frac{\mathbf{f_o}}{\mathbf{f_e}}\left(\mathbf{1} + \frac{\mathbf{f_e}}{\mathbf{D}}\right)$$

**NOTE:**

The **least distance of distinct vision** is the distance from the eye to a point where the eye can see an object in greatest detail. This distance is about **25 cm** for a normal adult eye but less for younger people.

**DISADVANTAGES OF AN ASTRONOMICAL TELESCOPE**

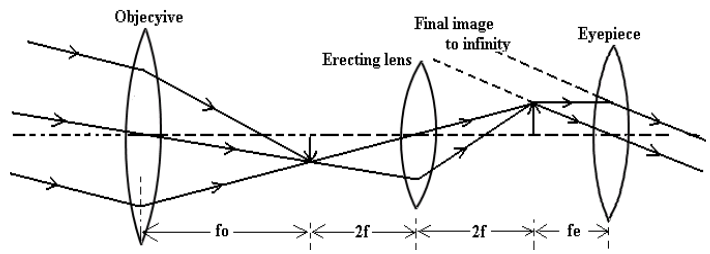
It forms an inverted final image.

**NOTE:**

The structure of an astronomical telescope can be modified to over come the above disadvantage by use of a terrestrial telescope which forms an erect image.

### TERRESTRIAL TELESCOPE

This consists of an intermediate erecting lens of focal length **f**, which is place between the objective lens and the eyepiece. The erecting lens should be a distance **2f** after the principal focus of the objective lens and a distance **2f** before the principal focus of the eyepiece. The objective lens forms a real inverted image of a distant object at its focal point $\mathbf{F_o}$. This act as a real object for the erecting lens which forms a real erect image of the same size as the inverted image formed by the objective.




### ADVANTAGE OF A TERRESTRIAL TELESCOPE

It forms an erect final image.

### DISADVANTAGES OF A TERRESTRIAL TELESCOPE

**(i)** It is bulky since its length is increased by **4f** compared with an astronomical telescope.

**(ii)** It reduces the intensity of light emerging through the eyepiece. This is due to light losses at several lens surfaces.

**EXAMPLES**:

**1.** An astronomical telescope has an objective and an eyepiece of focal length **100cm** and **5cm** respectively.

**(a)** Find the angular magnification of the telescope if arranged in normal adjustment.

**(b)** If the lenses are arranged in such a way that the final image is formed at **25cm** from the eyepiece, calculate the**:**

**(i)** angular magnification of the telescope in this setting.

**(ii)** separation of the objective and eyepiece.

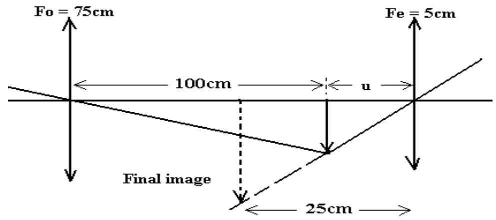
**Solution:**

**(a)** In normal adjustment, **magnifying power** $\mathbf{M} = \frac{f_o}{f_e} = \frac{100}{5} = \mathbf{20}$

**(b) (i)** With the final image at near point**, magnifying power** $\mathbf{M} = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$

$$\Rightarrow \quad \mathbf{M} = \frac{100}{5}\left(1 + \frac{5}{25}\right) = \mathbf{24}$$

**(ii)**




**The required lens separation** = $\mathbf{f_o + u}$

Consider the action of the eyepiece

v = -25cm and $f_e$ = 5cm

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ gives

$$\mathbf{u} = \frac{\mathbf{f \cdot v}}{\mathbf{v - f}} = \frac{5 \times -25}{^{-}25 - 5} = \mathbf{3 \cdot 57cm}$$

⇒ **The lens separation** = $\mathbf{f_o + u}$ = (100 + 3·57) cm = **103·57cm**

**2.** The objective of an astronomical telescope in normal adjustment has a diameter of **12cm** and focal length of **80cm**.

**(a)** If the eyepiece has a focal length of **5cm,** find the:

**(i)** magnifying power of the telescope in this setting.

**(ii)** Position of the eye-ring

**(iii)** diameter of the eye-ring

**(b)**State the advantage of placing the eye at the eye ring.

**Solution**

**(a) (i)** In normal adjustment, **magnifying power** $\mathbf{M} = \frac{f_o}{f_e} = \frac{80}{5} = \mathbf{16}$

**(ii)** Consider the action of the eyepiece

u = $\mathbf{f_o}$ + $\mathbf{f_e}$ = (80 + 5) = 85cm and $f_e$ = 5cm

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ gives

$$\mathbf{v} = \frac{\mathbf{f \cdot u}}{\mathbf{u - f}} = \frac{5 \times 85}{85 - 5} = \mathbf{5 \cdot 3125cm}$$

∴ The eye-ring is **5·3125cm** from the eyepiece

**(ii)** In normal adjustment, $\frac{\textbf{objective diameter}}{\textbf{eye - ring diameter}} = \frac{\mathbf{f_o}}{\mathbf{f_e}}$

$$\Rightarrow \quad \frac{18}{\text{eye - ring diameter}} = \frac{80}{5}$$

$$\therefore \quad \textbf{The diameter of the eye - ring} = \mathbf{1 \cdot 125cm}$$

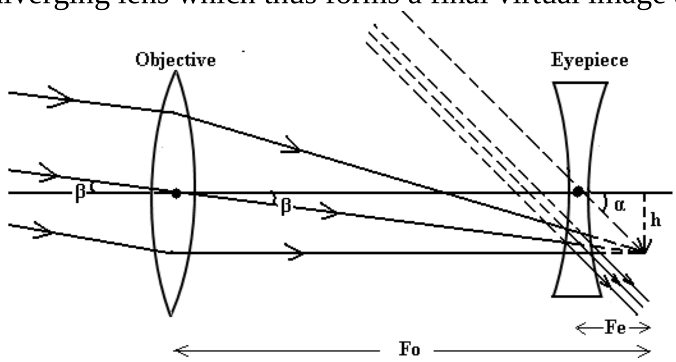
**(iii)** The eye placed at the eye ring has a wide field of view since most of the light entering the objective passes through the eye ring.

**GALILEAN TELESCOPE:**

This telescope provides an erect image of a distant object with the aid of an objective which is a converging lens of long focal length and an eyepiece which is a diverging lens of short focal length.

**GALILEAN TELESCOPE IN NORMAL ADJUSTMENT**

A converging lens is arranged coaxially with a diverging lens such that their focal points are at the same point. The converging lens forms a real image of a distant object at its focal point $\mathbf{F_o}$ situated exactly at the principal focus $\mathbf{F_e}$ of the diverging lens. This image formed acts as a virtual object for the diverging lens which thus forms a final virtual image at infinity as shown.




Let **h** be the height of the intermediate image formed at $\mathbf{F_o}$

$$M = \frac{\alpha}{\beta}$$

$$\boldsymbol{\alpha} \approx \tan\alpha = \mathbf{h}/\mathbf{f_e} \quad \text{and} \quad \boldsymbol{\beta} \approx \tan\beta = \mathbf{h}/\mathbf{f_o}$$

$$\Rightarrow \qquad M = \frac{h/f_e}{h/f_o}$$

$$\therefore \qquad \mathbf{M} = \frac{\mathbf{f_o}}{\mathbf{f_e}}$$

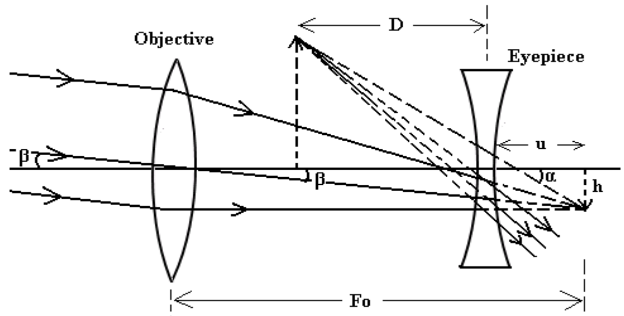
**NOTE:**

**(i)** From above, the distance between the two lenses is $(\mathbf{f_o - f_e})$

**(ii)** The image of the objective in the eyepiece (concave lens) is virtual and corresponds to a position between the two lenses. Therefore the eye-ring is inaccessible to the eye. However the best position of the eye in this case is as close as possible to the eyepiece and consequently the field of view for the telescope is very limited.

**GELILEAN TELESCOPE WITH FINAL IMAGE AT NEAR POINT**

A converging lens arranged coaxially with a diverging lens forms a real image of a distant object at its focal point $\mathbf{F_o}$ situated a distance **u** beyond the diverging lens. This image formed acts as a virtual object for the diverging lens which thus forms a final erect virtual image between the converging lens and the diverging lens at an image distance **D** as shown.




Let h be the height of the image formed at Fo.

Angular magnification $M = \frac{\alpha}{\beta}$

$\alpha$ and $\beta$ are small angles such that $\alpha \approx \tan\alpha = \mathbf{h}/\mathbf{u}$ and $\beta \approx \tan\beta = \mathbf{h}/\mathbf{f_o}$

$$\Rightarrow \qquad M = \frac{h/u}{h/f_o}$$

$$\therefore \qquad \mathbf{M} = \frac{\mathbf{f_o}}{\mathbf{u}} \quad \text{--------------------(i)}$$

Consider the action of the eyepiece

$\mathbf{v} = -\mathbf{D}$ and $\mathbf{f} = -\mathbf{f_e}$

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ gives

$$\mathbf{u} = \frac{f \cdot v}{v - f} = \frac{-f_e \times -D}{-D - {}^{-}f_e} = \frac{\mathbf{f_e D}}{\mathbf{f_e - D}}$$

$$\mathbf{M} = \frac{f_o}{f_e D / (f_e - D)} = \frac{\mathbf{f_o(f - D)}}{\mathbf{f_e D}}$$

$$\Rightarrow \qquad \mathbf{M} = \frac{\mathbf{f_o}}{\mathbf{f_e}}\left(\frac{\mathbf{f_e}}{\mathbf{D}} - \mathbf{1}\right)$$

**NOTE:**

**(i)**There is no need to consider the signs of $\mathbf{f}_e$ and **D** while using the above expression. **(ii)** The negative sign associated with the calculated magnifying power in this case is always omitted while stating the final result.

**ADVANTAGES OF A GALLEAN TEELESCOPE**

It forms a final erect image.

It is shorter than the terrestrial and astronomical telescopes.

**DISADVANTAGES OF A GALILEAN TELESCOPE**

It has a small field of view.

It has a virtual eye ring not accessible to the observer.

**EXAMPLE**

**1.** A Galilean telescope has a convex lens of focal length **50cm** and a diverging lens of focal length **5cm.**

**(a)** Find the angular magnification of the telescope if arranged in normal adjustment.

**(b)** If the lenses are arranged in such a way that the final image is formed at **25cm** from the eyepiece, calculate the**:**

**(i)** angular magnification of the telescope in this setting.

**(ii)** separation of the objective and eyepiece

**Solution**

**(a) (i)** In normal adjustment, **magnifying power** $\mathbf{M} = \frac{f_o}{f_e} = \frac{50}{5} = \mathbf{10}$

**(b) (i)** With the final image at near point**, magnifying power** $\mathbf{M} = \frac{f_o}{f_e}\left(\frac{f_e}{D} - 1\right)$

$$\Rightarrow \quad \mathbf{M} = \frac{50}{5}\left(\frac{5}{25} - 1\right) = \mathbf{-8}$$

**Thus the required angular magnification is 8**

**(ii)**

**The lens separation** = $\mathbf{f_o - u}$

Consider the action of the eyepiece

$v = -25cm$ and $f_e = -5cm$

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ gives

$$\mathbf{u} = \frac{\mathbf{f \cdot v}}{\mathbf{v - f}} = \frac{-5 \times -25}{-25 - {}^{-}5} = \mathbf{-8 \cdot 33cm}$$

$\Rightarrow$ The image of a distant object formed by the objective is **8·33cm** beyond the eyepiece.

**The required lens separation** = $\mathbf{f_o - u}$ = (50 – 8·33) cm = 41**·67cm**

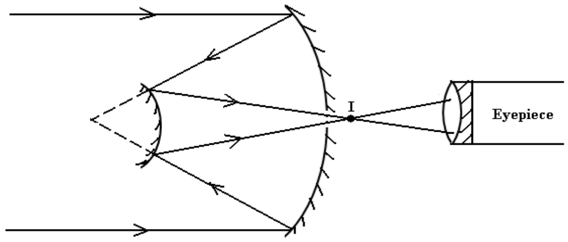
**REFLECTING ASTRONOMICAL TELESCOPE**

The objective of a reflecting telescope is a concave mirror with long focal length.
There are three types of reflector telescopes namely:

**(i)** Cassegrain Reflector Telescope
**(ii)** Newton Reflector Telescope
**(iii)** Coude Reflector Telescope

**CASEGRAIN REFLECTOR TELESCOPE IN NORMAL ADJUSTMENT**

The objective consists of a concave mirror with a long focal length. Light from a distant object is reflected first at a concave mirror and then at a small convex mirror to form a real image **I** at a hole situated at the pole of the concave mirror. The eyepiece is set such that **I** coincide with its principal focus thus forming a magnified virtual image at infinity as shown.




NB:

(i) The magnifying power of this telescope in normal adjustment is given by

$$M = \frac{Fo}{Fe}\left(\frac{v}{u}\right)$$

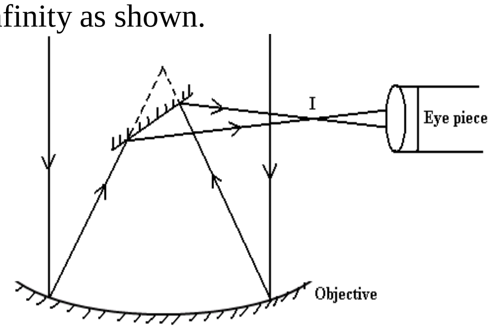
Where $\left(\frac{v}{U}\right)$ is the linear magnification produced

By two convex mirrors Fo and Fe are the focal length of the objective and the eyepiece respectively.

(ii) In the absence of a convex mirror, the concave mirror would form the image of a distant object at its focal point Fo

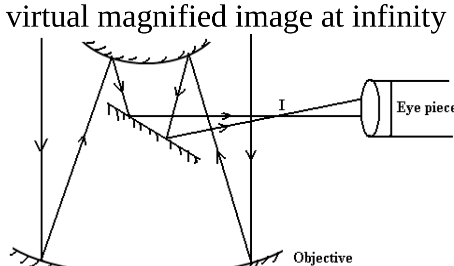
**NEWTON REFLECTOR TELESCOPE IN NORMAL ADJUSTMENT**

The objective consists of a concave mirror of long focal length. Light from a distant object is reflected first at the concave mirror and then at a small slanting plane mirror to form a real image at **I**. The eye piece is set such that **I** coincide with its principal focus thus forming a magnified virtual image at infinity as shown.




**COUDE REFLECTOR TELESCOPE IN NORMAL ADJUSTMENT**

The objective consists of a concave mirror of long focal length. Light from a distant object is reflected first at a concave mirror and then at a small convex mirror which then reflects it on to a slanting plane mirror to form a real image at I. The eyepiece is set such that **I** coincide with its principal focus thus forming a virtual magnified image at infinity as shown.




**ADVANTAGES OF REFLECTING TELESCOPES**

**(i)** There is no chromatic aberration since no refraction occurs at the objective

**(ii)** There is no spherical aberration since a paraboloidal mirror is used.

**(iii)** It is cheaper to construct since only one surface requires grinding.

**(iv)** When curved mirrors of large diameter are used, a greater resolving power is obtained.

**EXAMPLE**

A distant object subtending $\mathbf{6 \times 10^{-3}}$ radian is viewed with a reflecting telescope whose objective is a concave mirror of focal length **15m**. The reflected light is intercepted by a convex mirror placed **12cm** from the pole of the objective where there is a hole. The image is viewed with a convex lens of focal length **5cm** used as a magnifying glass producing a final image at infinity.

**(a)** Draw a ray diagram for this arrangement

**(b)** Calculate the:

**(i)** diameter of the real image that would be formed at the focus of the concave mirror.

**(ii)** diameter of the image formed at the pole of the concave mirror.

**(iii)** angular magnification for the arrangement.

**EXERCISE;**

**1. (i)** Define the term **visual angle** as applied to optical systems**.**

**(ii)** Explain why the farthest vertical pole in line with others of equal height looks shorter.

**2. (i)** Define the term **angular magnification** of an optical instrument.

**(ii)** State the reason why the focal length of the objective of a telescope is always much longer than that of the eyepiece.

**(iii)** What is meant by the term **normal adjustment** as applied to a telescope?

**(iv).** Draw a ray diagram to show the action of an astronomical telescope in normal adjustment, and derive an expression for its magnifying power in terms of the focal lengths $\mathbf{f_o}$ and $\mathbf{f_e}$ of the objective and eyepiece respectively.

**(v)** Calculate the separation of the eyepiece and objective of an astronomical telescope in normal adjustment whose magnifying power is **20** and its eyepiece has a focal length of **5cm.**

**[Answer: 105cm]**

**3 (i)** What is meant by the term **exit pupil** as applied to a telescope?

**(ii)** What is the significance of the eye-ring of an astronomical telescope?

**(iii)** With the aid of a ray diagram, show that for an astronomical telescope in normal adjustment having its objective lens and an eyepiece of focal lengths $\mathbf{f_o}$ and $\mathbf{f_e}$ respectively, $\dfrac{\textbf{diameter of objective}}{\textbf{diameter of eye-ring}} = \dfrac{\mathbf{f_o}}{\mathbf{f_e}}$.

**(iv)** Calculate the distance of the eye-ring from the eyepiece of an astronomical telescope in normal adjustment whose objective and eyepiece have focal lengths of **80cm** and **10cm** respectively.

**[Answer: 11·25cm]**

**(v).** Draw a ray diagram to show the formation of the final image by an astronomical telescope at near point, and derive an expression for its magnifying power.

**(vi)** State the disadvantage of using an astronomical telescope when viewing distant objects on earth. Describe how an astronomical telescope can be modified to overcome this disadvantage.

**4. (i)** The objective and eyepiece of an astronomical telescope have focal lengths $\mathbf{f_o}$ and $\mathbf{f_e}$ respectively. Derive an expression for the magnifying power of this telescope if the final image is a distance **D** in front of the eyepiece.

**(ii)** The objective and eyepiece of an astronomical telescope have focal lengths of **80cm** and **5cm** respectively. Calculate the magnifying power of this telescope and separation of its two lenses if arranged in normal adjustment.

**[Answer: 16, 85cm]**

**(iii)** The objective and eyepiece of an astronomical telescope have focal lengths of **75cm** and **2·5cm** respectively. Calculate the magnifying power of this telescope and separation of its two lenses if the final image is a distance **25cm** in front of the eyepiece.

**[Answer: 33, 77·273cm]**

**(iv)** An astronomical telescope has an objective and an eyepiece of focal lengths **80cm a**nd **5cm** respectively. If the lenses are arranged in such a way that the final image of a distant object which subtends an angle of **0·6°** at the objective is formed at a distance of **25cm** in front of the eyepiece, calculate the:

**(a)** angular magnification and the separation of the lenses in this setting

**(b)** size of the final image seen.

**[Answer: (a) 19·2, 84·17cm (b) 5·03cm ]**

**(v)** Draw a ray diagram to show the action of a terrestrial telescope in normal adjustment. List one advantage and one disadvantage of this type of telescope.

**5. (i)** Draw a ray diagram to show the action of a Galilean telescope in normal adjustment, and derive an expression for its magnifying power in terms of the focal lengths $\mathbf{f_o}$ and $\mathbf{f_e}$ of the objective and eyepiece respectively.

**(ii)** Calculate the separation of the eyepiece and objective of a Galilean telescope in normal adjustment whose magnifying power is **20** and its eyepiece has a focal length of **5cm.**

**[Answer: 95cm]**

**(iii)** Draw a ray diagram to show the formation of the final image by a Galilean telescope at near point, and derive an expression for its magnifying power.

**(iv)** The objective and eyepiece of a Galilean telescope have focal lengths $\mathbf{f_o}$ and $\mathbf{f_e}$ respectively. Derive an expression for the magnifying power of this telescope if the final image is a distance **D** in front of the eyepiece.

**(v)** With the aid of a ray diagram, describe the action of a telescope made up of a converging and a diverging lens when used in normal adjustment. List one advantage and one disadvantage of this type of telescope.

**6. (i)** A convex lens of focal length **60cm** is arranged co-axially with a diverging lens of focal length **5cm,** to view a distant star.

**(a)** If the final image is at infinity, draw a ray diagram to show the formation of the Image of the star.

**(b)** Calculate the angular magnification obtained if the image of the star is formed at a distance of **25cm** in front of the eyepiece.

**(c)** List one advantage and one disadvantage of this type of arrangement over an astronomical telescope.

**(ii)** With the aid of a ray diagram, describe the structure and action of a reflecting telescope in normal adjustment.

**(iii)** State **two** advantages of a reflecting telescope over a refracting telescope.

**[Answer: (i) (b) 9·6]**

**7.** A small convex mirror is placed **100cm** from the pole and on the axis of a large concave mirror of radius of curvature **320cm.** The position of the convex mirror is such that a real image of a distant object is formed in the plane of a hole drilled through the concave mirror at its pole.

**(a) (i)** Draw a ray diagram to show how a convex mirror forms an image of a non-axial point of a distant object

**(ii)** Suggest a practical application for the arrangement of mirrors in **a (i)** above**.**

**(iii)** Calculate the radius of curvature of the convex mirror

**(b)** If the distant object subtends an angle of $\mathbf{3 \times 10^{-3}}$ **radians** at the pole of the concave mirror, calculate the

**(i)** size of the real image that would have been formed at the focus of the concave mirror.

**(ii)** size of the image formed by the convex mirror

**[ Answers: (a) (iii) 150cm (b) (i) 0·48cm (ii) 0·8cm ]**

**MICROSCOPES**

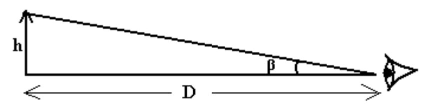
A microscope is an instrument used for viewing near objects. In normal use, the image formed by the microscope is usually at the least distance of distinct vision, **D,** from the eye.

**ANGULAR MAGNIFICATION OF A MICROSCOPE**:

This is the ratio of the angle subtended at the eye by the image at near point when the microscope is used to the angle subtended at the unaided eye by the object at near point.

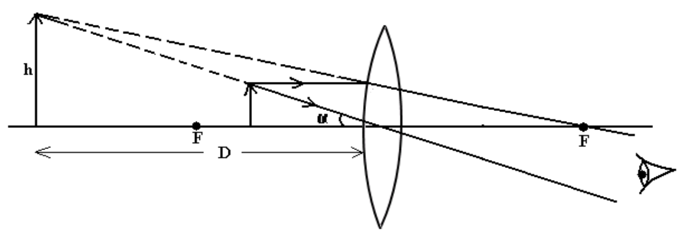
**SIMPLE MICROSCOPE OR MAGNIFYING GLASS IN NORMAL USE ADJUSTMENT**

Before using a microscope, the object is first viewed at the near point of the eye by unaided eye as shown:




Let an object of height **h** viewed at the near point subtend an angle **β** at the unaided eye. Angle **β** is very small such that $\boldsymbol{\beta} \approx \tan\beta = \frac{\mathbf{h}}{\mathbf{D}}$

A simple microscope in normal adjustment consists of a converging lens set in such a way that it forms a virtual magnified erect image of an object placed between the principal focus and the optical centre of the lens at the least distance of distinct vision as shown.




Let **$h_1$** be the height of the image formed at the near point.

Angle **α** is very small such that $\boldsymbol{\alpha} \approx \tan\alpha = \frac{\mathbf{h_1}}{\mathbf{D}}$

Angular magnification of this telescope is given by

$$\mathbf{M} = \frac{\alpha}{\beta}$$

$$\Rightarrow \quad \mathbf{M} = \frac{h_1/D}{h/D} = \frac{\mathbf{h_1}}{\mathbf{h}}$$

But the ratio $\frac{\mathbf{h_1}}{\mathbf{h}}$ is the linear magnification **m** produced by the lens

In this case **M** = **m** where $\mathbf{m} = \frac{v}{f} - 1$ and v = -D

**Thus** $\mathbf{M} = \frac{\mathbf{-D}}{\mathbf{f}} \mathbf{- 1}$**,** where **f** is the focal length of the lens

**EXAMPLE:**

Calculate the angular magnification produced by a magnifying glass of focal length **5cm** adjusted such that an image is formed at a distance of **25cm** in front of it.

**Solution:**

Using $\mathbf{M} = \frac{\mathbf{-D}}{\mathbf{f}} \mathbf{- 1}$ where $\mathbf{D = 25cm}$

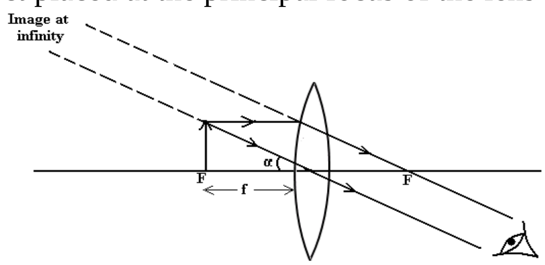
$$\therefore \quad \mathbf{M} = \frac{-25}{5} - 1 = \mathbf{-6}$$

Thus the required angular magnification is **6**

### SIMPLE MICROSCOPE WITH FINAL IMAGE AT INFINITY

Let an object of height **h** viewed at the near point subtend an angle $\boldsymbol{\beta}$ at the unaided eye. Angle $\boldsymbol{\beta}$ is very small such that $\boldsymbol{\beta} \approx \tan\beta = \frac{\mathbf{h}}{\mathbf{D}}$

This simple microscope consists of a converging lens which forms an erect virtual magnified image at infinity of an object placed at the principal focus of the lens as shown.



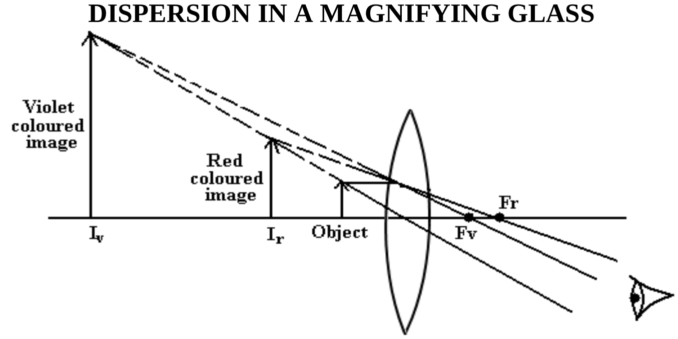

Angle $\boldsymbol{\alpha}$ is very small such that $\boldsymbol{\alpha} \approx \tan\alpha = \frac{\mathbf{h}}{\mathbf{f}}$

Thus magnifying power $\mathbf{M} = \frac{\alpha}{\beta}$

$$\Rightarrow \quad \mathbf{M} = \frac{h/f}{h/D}$$

**Thus** $\mathbf{M} = \frac{\mathbf{D}}{\mathbf{f}}$ **,** where **f** is the focal length of the lens

### DISPERSION IN A MAGNIFYING GLASS



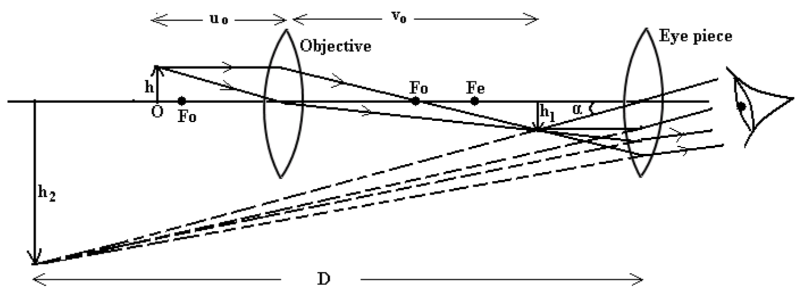

When an object **O** is viewed through a converging lens used as a magnifying glass, various coloured virtual images corresponding to say red and violet rays are formed at slightly different positions $\mathbf{I_r}$ and $\mathbf{I_v}$ respectively as shown. These images subtend the same angle at the eye and therefore appear superimposed. Thus the virtual image seen
in a simple microscope is almost free from chromatic aberration.

**NOTE:**

A real image formed by the lens as explained in the previous section has chromatic aberration.

**COMPOUND MICROSCOPE IN NORMAL ADJUSTMENT**

A compound microscope consists of two converging lenses of short focal lengths. This enables a high angular magnification to be obtained.
In normal adjustment, the objective of a compound microscope forms a real inverted image of the object at a point distance less than Fe from the eyepiece. This intermediate image formed acts as a real object for the eye piece which thus forms a virtual magnified image at a distance of distinct vision from the eye piece as shown.




Angle $\alpha$ is very small such that $\alpha \approx \tan\alpha = \frac{h_2}{D}$

Let an object of height **h** viewed at the near point subtend an angle $\beta$ at the unaided eye. Angle $\beta$ is very small such that $\beta \approx \tan\beta = \frac{h}{D}$

Thus magnifying power $M = \frac{\alpha}{\beta}$

$$\Rightarrow \qquad M = \frac{h_2/D}{h/D} = \frac{h_2}{h}$$

$\therefore \qquad M = \frac{h_2}{h_1} \times \frac{h_1}{h}$ where $h_1$ is the height of the intermediate image.

But the ratio $\frac{h_2}{h_1}$ and $\frac{h_1}{h}$ are the linear magnifications produced by the eye piece and objective respectively.

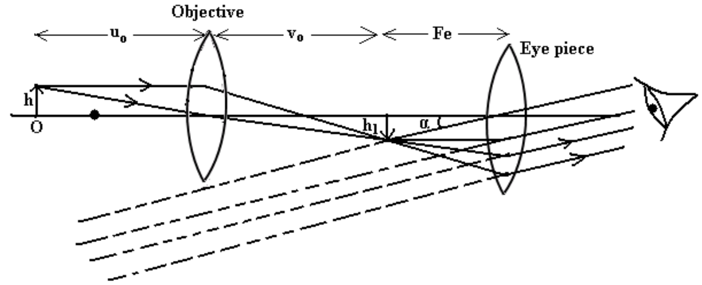
In this case $\frac{h_2}{h_1} = \frac{-D}{f_e} - 1$ and $\frac{h_1}{h} = \frac{v_o}{f_o} - 1$

**Thus** $M = -\left(\frac{D}{f_e} + 1\right)\left(\frac{v_o}{f_o} - 1\right)$

**NOTE:**
The above expression clearly shows that if $f_o$ and $f_e$ are small then a high angular magnification is obtained.

**COMPOUND MICROSCOPE WITH FINAL IMAGE AT INFINITY**

The objective forms a real inverted image of the object at the principle focus Fe of the eye piece which thus forms a final virtual magnified image at infinity as shown.




Angle $\boldsymbol{\alpha}$ is very small such that $\boldsymbol{\alpha} \approx \tan\alpha = \dfrac{\mathbf{h_1}}{\mathbf{f_e}}$

Let an object of height **h** viewed at the near point subtend an angle $\boldsymbol{\beta}$ at the unaided eye. Angle $\boldsymbol{\beta}$ is very small such that $\boldsymbol{\beta} \approx \tan\beta = \dfrac{\mathbf{h}}{\mathbf{D}}$

Thus magnifying power $\mathbf{M} = \dfrac{\alpha}{\beta}$

$$\Rightarrow \qquad \mathbf{M} = \frac{h_1/f_e}{h/D} = \frac{\mathbf{h_1}}{\mathbf{h}} \times \frac{\mathbf{D}}{\mathbf{f_e}}$$

But the ratio $\dfrac{\mathbf{h_1}}{\mathbf{h}}$ is the linear magnification produced by the objective.

In this case $\dfrac{\mathbf{h_1}}{\mathbf{h}} = \dfrac{\mathbf{v_o}}{\mathbf{f_o}} - \mathbf{1}$

$$\textbf{Thus } \mathbf{M} = \left(\frac{\mathbf{v_o}}{\mathbf{f_o}} - \mathbf{1}\right)\frac{\mathbf{D}}{\mathbf{f_e}}$$

### DIFFERENCES BETWEEN A COMPOUD MICROSCOPE AND A TELESCOPE

**(i)** Compound microscopes are used for viewing near objects while telescopes are used for viewing distant objects.

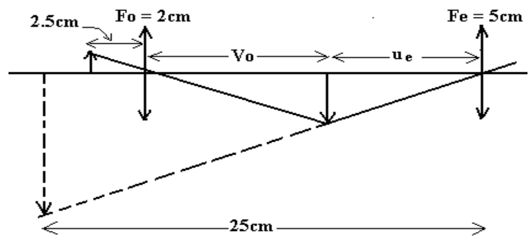
**(ii)** The objective of a compound microscope has got a short focal length while for a telescope it has got a long focal length.

**(iii)** In normal use, a compound microscope forms the final image at near point while for a telescope the final image is formed at infinity.

**EXAMPLES:**

**1.** The objective of a compound microscope has a focal length of **2cm** while the eyepiece has a focal length of **5cm**. An object is placed at a distance of **2.5cm** in front of the objective. The distance of the eyepiece from the objective is adjusted so that the final image is **25cm** in front of the eyepiece. Find the distance between the lenses and the magnifying power of the microscope.

**Solution:**




Consider the action of the eyepiece

$v_e$ = -25cm and $f_e$ = 5cm

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ gives

$$\mathbf{u_e} = \frac{f_e \cdot v_e}{v_e - f_e} = \frac{5 \times -25}{-25 - 5} = \mathbf{4 \cdot 167cm}$$

Consider the action of the objective

$u_o$ = 2·5cm and $f_o$ = 2cm

Using the lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ gives

$$\mathbf{v_o} = \frac{f_o \cdot u_o}{u_o - f_o} = \frac{2 \times 2 \cdot 5}{2 \cdot 5 - 2} = \mathbf{10cm}$$

$\therefore$ **The required lens Separation** = $\mathbf{v_o + u_e}$ = (10 + 4·2) cm = **14·2cm**

The required magnifying power $\mathbf{M} = -\left(\frac{\mathbf{D}}{\mathbf{f_e}} + \mathbf{1}\right)\left(\frac{\mathbf{v_o}}{\mathbf{f_o}} - \mathbf{1}\right)$ where **D = 25cm**

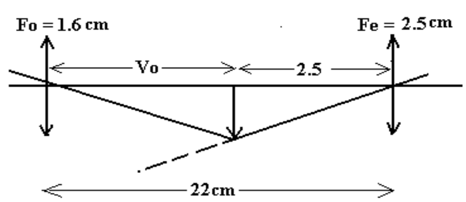
$$\Rightarrow \qquad \mathbf{M} = -\left(\frac{25}{5} + 1\right)\left(\frac{10}{2} - 1\right)$$

$\therefore$ **M = -24**

Thus the required magnifying power **M = 24**

**2.** A compound microscope has an eyepiece of focal length **2·5cm** and an objective of focal length **1·6cm**. If the distance between the objective and the eye piece is **22cm,** calculate the magnifying power produced when the object is at infinity.

Solution




**ANALYSIS:**

For the image to be at infinity, the object must be at the focal point of the eyepiece

Thus the image distance in the objective = (22 – 2·5) cm = **19·5cm**

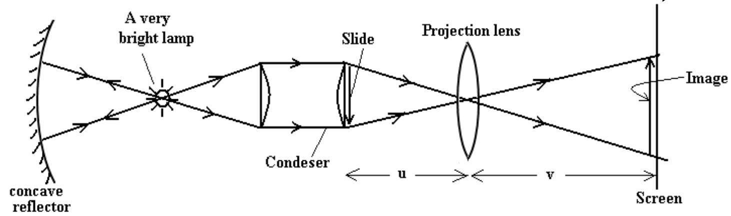
The required magnifying power $\mathbf{M} = \left(\frac{\mathbf{v_o}}{\mathbf{f_o}} - \mathbf{1}\right)\frac{\mathbf{D}}{\mathbf{f_e}}$ where **D = -25cm**

$$\Rightarrow \qquad \mathbf{M} = \left(\frac{19\cdot5}{1\cdot6} - 1\right) \times \frac{-25}{2\cdot5}$$

$\therefore$ **M** = -111·875

Thus the required magnifying power **M = 111·875**

**A SLIDE PROJECTOR (PROJECTION LANTERN)**




A strong source of light is placed at the centre of curvature of the concave reflector. Light from the source is reflected back onto the condenser which concentrates it onto the slide. The projection lens forms a magnified real image of the slide on the screen.

**NOTE:**

**(i)** A projector is used for showing a magnified image of the film or slides on the screen.

**(ii)** The film is the region where the slide is to be placed.

**(iii)** The magnification of the slide is given by $\mathbf{m} = \frac{\mathbf{v}}{\mathbf{u}}$ OR $\mathbf{m} = \frac{\mathbf{v}}{\mathbf{f}} - \mathbf{1}$, where **v** and **u** are the respective screen and slide distances from the projection lens of focal length.**f.**

**(iv)** The above expression clearly shows that a high magnification can be produced by using a projection lens of short focal length **f** compared to distance **v**.

**(v)** For an enlarged image, $\textbf{Image Area} = (\textbf{Linear scale factor})^2 \times \textbf{Object Area}$

But Linear scale factor = magnification.

$$\Rightarrow \text{Magnification } \mathbf{m} = \sqrt{\frac{\textbf{Image Area}}{\textbf{Object Area}}}$$

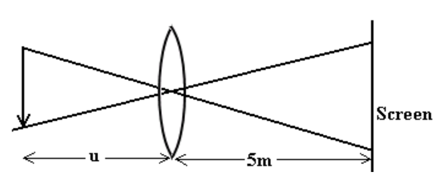
**EXAMPLE:**

**1.** A projector produces an image of area **1·2m** x **1·8m** onto a screen placed **5m** from the projector lens. If the area of the object slide is **2·4cm** x **3·6cm,** Calculate the:

**(i)** focal length of the projection lens.

**(ii)** distance of the slide from the lens.

**Solution:**




**(i)** Image area = 1·2m x 1·8m = $\mathbf{2\cdot16m^2}$

Object area = 2·4cm x 3·6cm = 8·64cm$^2$ = $\mathbf{8\cdot64 \times 10^{-4}m^2}$

Image distance **v = 5m**

Using the relation $\mathbf{m} = \sqrt{\dfrac{\textbf{Image Area}}{\textbf{ObjectArea}}} = \dfrac{\mathbf{v}}{\mathbf{f}} \mathbf{-1} = \dfrac{\mathbf{v}}{\mathbf{u}}$ gives

$$m = \sqrt{\frac{2\cdot16}{8\cdot64\times10^{-4}}} = \mathbf{50}$$

$$\Rightarrow \quad 50 = \frac{5}{f} - 1$$

$\therefore$ The required focal length **f = 0·098m OR f = 9·8cm**

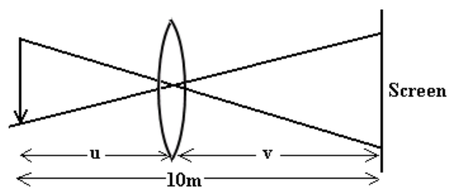
**(ii)** Using the relation $\mathbf{m} = \dfrac{\mathbf{v}}{\mathbf{u}}$ gives

$$50 = \frac{5}{u}$$

$\Rightarrow$ The distance of the slide from the lens **u = 0·1m OR u = 10cm**

**2.** A projector is required to project slides which are **7·5cm** square onto a screen which is **4·2m** square. If the distance between the slide and the screen is **10m,** what focal length of the projection lens would you consider more suitable.

**Solution**




Object area = 75 cm square = 7·5 x 7·5cm$^2$ = **7·5 x 7·5 x 10$^{-4}$m$^2$**

Image area = 4·2 m square = **4·2 x 4·2m$^2$**

Using the relation $\mathbf{m} = \sqrt{\dfrac{\textbf{Image Area}}{\textbf{Object Area}}} = \dfrac{\mathbf{v}}{\mathbf{f}} \mathbf{-1} = \dfrac{\mathbf{v}}{\mathbf{u}}$ gives

$$\mathbf{m} = \sqrt{\frac{4\cdot2\times4\cdot2}{7\cdot5\times7\cdot5\times10^{-4}}} = \mathbf{56}$$

$$\Rightarrow \quad 56 = \frac{v}{u}$$

$\therefore$ **v = 56u ----------------------------------(i)**

**But u + v = 10m -----------------------------(ii)**

Substituting equation **(i)** in **(ii)** gives

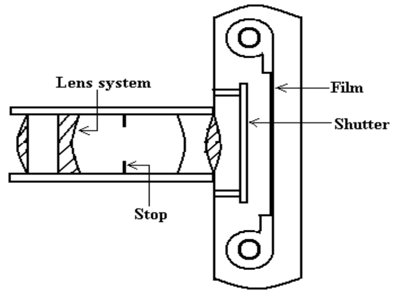
**u = 0·1754m** and **v = 9·8246m**

But $\mathbf{m} = \dfrac{\mathbf{v}}{\mathbf{f}} \mathbf{-1}$

$$\Rightarrow \quad \mathbf{f} = \frac{v}{m+1} = \frac{9\cdot8246}{56+1} = \mathbf{0\cdot1724m}$$

$\therefore$ The required focal length **f = 0·1724m OR f = 17·24cm**

### PHOTOGRAPHIC CAMERA




The camera lens system focuses light from an object onto the film. The film is where the image is formed. The shutter cuts off light where necessary. The stop regulates amount of light incident on the film.

**NOTE:**

**(i)** Chromatic aberration in a photographic camera is minimized by use of an achromatic combination of two lenses one convex and the other concave to form an achromatic doublet.

**(ii)** Spherical aberration in a photographic camera is minimized by use of a shield with a small hole in the middle and lenses of small aperture.

### PRISM BINOCULARS

Two right-angled isosceles prisms are placed between the objective and the eyepiece both of them being convex lenses. One prism is placed with its refracting edge vertical, while the other with its refracting edge horizontal. The first prism turns round the image formed by the objective in a horizontal direction, while the second prism inverts this image in a vertical direction. The image now formed at the principal focus of the eyepiece is the same way up and the same way round as the original object. The eyepiece forms an erect virtual image at infinity.

### ADVANTAGES OF PRISM BINOCULARS

**(i)** It forms a final erect image
**(ii)** It has a wide field of view
**(iii)** It has a compacted size

**EXERCISE:**

**1. (i)** Define the term **angular magnification** as applied to a microscope.

**(ii)** What is meant by the term **normal adjustment** as applied to a microscope?

**(iii)** Describe with the aid of a ray diagram, the action of a magnifying glass in normal adjustment. Hence derive an expression for its magnifying power in terms of the focal length **f** of the magnifying lens.

**(iv)** Describe with the aid of a ray diagram, the action of a magnifying glass in forming the final image at infinity. Hence show that in this case magnifying power $\mathbf{M} = \frac{\mathbf{D}}{\mathbf{f}}$, where **f** is the focal length of the magnifying lens and **D** is the distance of most distinct vision.

**(v)** Explain why chromatic aberration is **not** observed in a simple microscope.

**2. (i)** State the reason why the focal lengths of the objective and eyepiece of a compound microscope are both small.

**(ii).** Describe with the aid of a ray diagram, the action of a compound microscope in normal adjustment. Hence derive an expression for its magnifying power in terms of the focal lengths $\mathbf{f_o}$ and $\mathbf{f_e}$ of the objective and eyepiece respectively.

**(iii).** Describe with the aid of a ray diagram, the action of a compound microscope in forming the final image at infinity. Hence derive an expression for its angular magnification in terms of the focal lengths $\mathbf{f_o}$ and $\mathbf{f_e}$ of the objective and eyepiece respectively

**(iv)** State three differences between compound microscopes and telescopes.

**3. (i)** The objective of a compound microscope has a focal length of **5cm** while the eyepiece has a focal length of **4cm**. If the distance between them is **20cm** the final image of an object placed in front of the objective is formed **25cm** in front of the eyepiece. Calculate the position of the object and the magnifying power of the microscope.

**(ii)** The objective of a compound microscope has a focal length of **4·2cm**. An object placed **6cm** in front of the objective gives rise to a final image at a distance of **24cm** in front of the eyepiece and in the plane of the object when viewed with the eye close to the eyepiece. Calculate the:

**(a)** separation of the lenses.

**(b)** focal length of the eyepiece.

**(c)** angular magnification of the microscope.

**[ Answers: (a) 18cm (b) 4·8cm (c) 14·6 ]**

**(iii)** An object of size **2·0mm** is placed **3·0cm** in front of the objective of a compound microscope. The objective of a compound microscope has a focal length of **2·5cm** while the eyepiece has a focal length of **5·0cm.** The microscope forms a virtual image of the object at the near point of the eye. Find the:

**(a)** size of the final image

**(b)** position of the eye-ring

**[ Answers: (a) 60mm (b) 6·85cm ]**

## STATIC ELECTRICITY.

**Definition:**

Static electricity is the electricity concerned with electric currents at rest. This electricity is usually produced by either friction or induction. The study of these charges, which are packets of electricity at rest is known as **electrostatics.** There are basically two types of charges i.e. **negative charge** and **positive charge.**

**Law of electrostatics:**

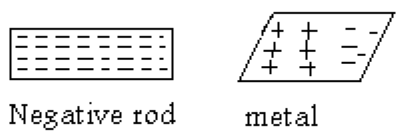
This states that opposite charges attract each other while like charges repel one another.

**Charging by friction.**

A fountain pen rubbed with a coat sleeve attracts pieces of papers, but a metal rubbed with the same coat sleeve does not attract pieces of paper. This shows that the pen has attained a charge but the metal has not. However, the same metal can be made charged by rubbing it with fur of silk when held using an insulator. Similarly a glass rubbed silk develops a charge as well as an ebonite rubbed with fur. All these charges developed are as a result of friction.
Conclusively, when two insulators, say A and B are rubbed together, they attain equal but opposite charges. This is because when A and B are rubbed together, their electrons gain kinetic energy and the one with low work function (A) will loose electrons to B with high work function. A therefore becomes positively charged while B becomes negatively charged. Since the number of electrons lost by A equals that of electrons gained by B, they attain equal but opposite charges. This can be verified using Ice pail experiment.

**Charging by induction.**

Induced charges are obtained when a neutral body attains charges by bringing a charged body near without contact between the two bodies. Here it is assumed that the uncharged body is a conductor.



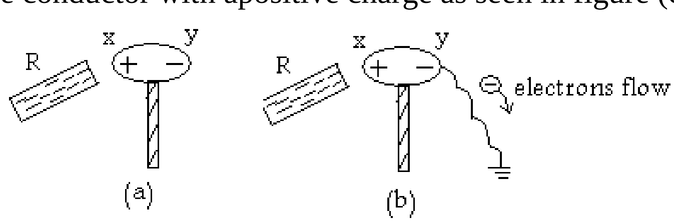

If a metal conductor is brought near a charged body such as a negatively charged rod as seen in the figure above, the negative charge on the rod repels the free electrons in the metal conductor to its far end and a positive charge is left at the near end of the metal. The negative charge of the rod therefore attracts the positive charge at the near end. This explains why neutral conductors are attracted by charged bodies when they are brought near each other.

**Inducing a permanent charge on a body.**

A negatively charged rod e.g. polythen rod (R) is brought near the conductor. The charge on the rod repels electrons to end y leaving an equal positive charge near end x as seen in figure (a) below

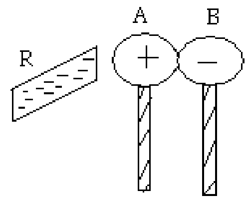
The conductor is earthed by touching it momentarily in the presence of the rod so that electrons repelled flow to the earth as shown in figure (b) below

The earthing is removed in the presence of the rod, and then the rod removed, which leaves the conductor with apositive charge as seen in figure (c) below



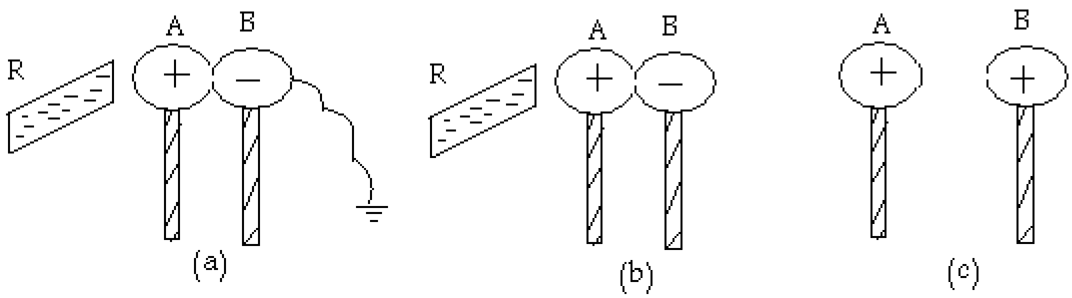

**NB:** If a positively charged rod is used, the conductor attains a negative charge.

**Inducing charges of different signs on two bodies simultaneously.**



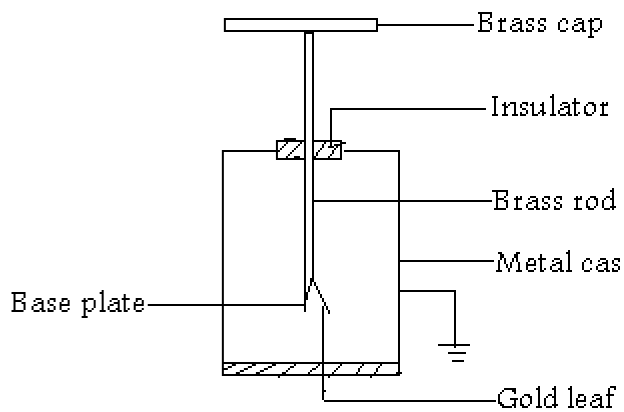

Two insulated spheres A and B are arranged as above. A negatively charged rod R is brought near A which leads to repulsion of free electrons from A to B. the two spheres are separated in the presence of the rod, which is also removed later. When A and B are tested, they are found to have positive and negative charges respectively.
**NB:** If a positively charged rod is used, A and B will attain negative and positive charges respectively.

**Inducing charges of the same sign on two bodies simultaneously**

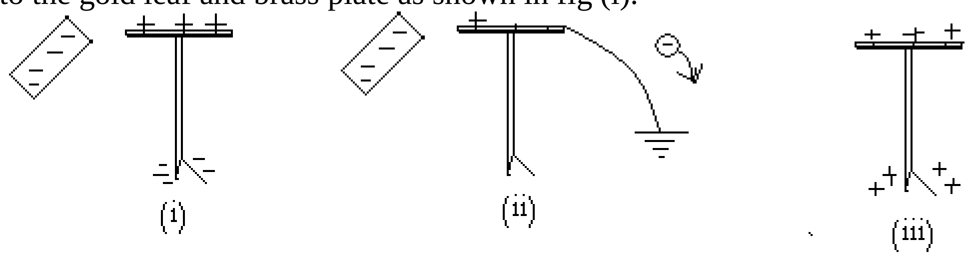



The two metal spheres A and B are mounted on insulators and placed in contact. A negatively charged rod is brought near A which induces a positive charge on A and a negative charge on B. by earthing B in the presence of the rod, electrons flow into the ground. The earthing connection is then broken and the rod removed. The positive charge distributes in the spheres and thus when they are separated and tested, they are found to have only positive charges.
**NB.** If a positively charged rod is used, the two spheres attain a negative charge.

**Induction and the gold leaf electroscope.**



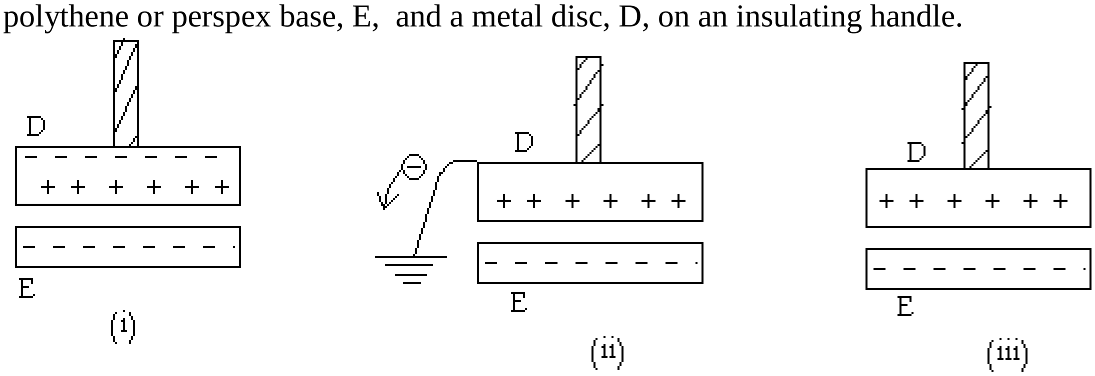

An electroscope can be charged similarly by induction. When a negatively charged rod is brought near the brass cap, it induces a positive charge on the cap and the electrons are repelled to the gold leaf and brass plate as shown in fig (i).




The electroscope is then earthed as in fig (ii). The electros move from the gold leaf and the plate to the ground.
When the earth is broken and the charging body removed as in fig (iii), the electroscope becomes positively charged.

## ELECTROPHORUS

This is a device that produces almost unlimited supply of charge by induction. It consists of a polythene or perspex base, E, and a metal disc, D, on an insulating handle.




The polythene is charged negatively by rubbing it vigorously with another material such as a duster. When the disc is laid upon it, it acquires induced positive charge after earthling it with a finger. Very little negative charge escapes from the polythene to the disc because the metal surface has few points of contact with the polythene which is also a poor conductor. On removing the disc, it has sufficient positive charge which can give audible and visible sparks.

The disc can be discharged and charged again repeatedly until the charge on the polythene has disappeared by leakage.

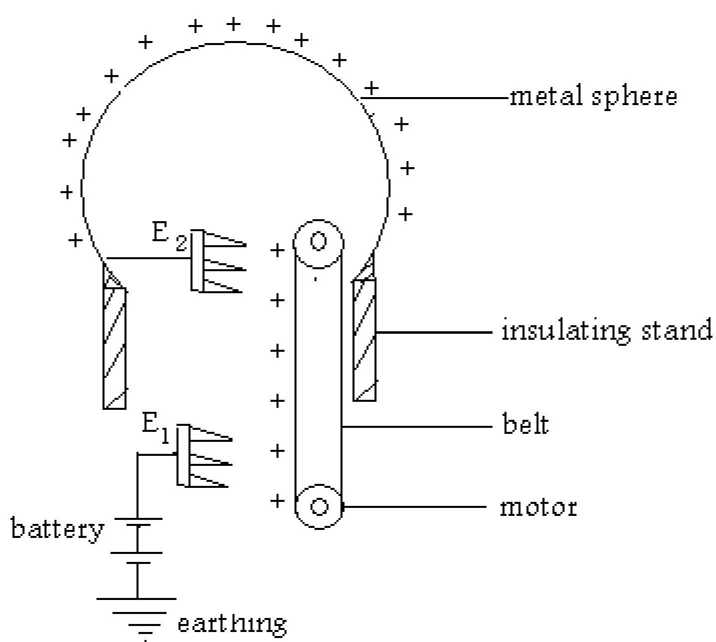
**Advantages of electrophorus.**

- Supply of charge is unlimited because the original charge is not carried away. (only a small charge could be transferred by contact leakage because the polythene is not a good conductor.)
- A great charge equal to the charging body can be concentrated in a small area if the area of the charging body can be made small.
- It acts as a device for converting mechanical energy to electrical energy as work is done in rising the disc against the attraction of the positive charge.

**The Van Da Graaff Generator.**

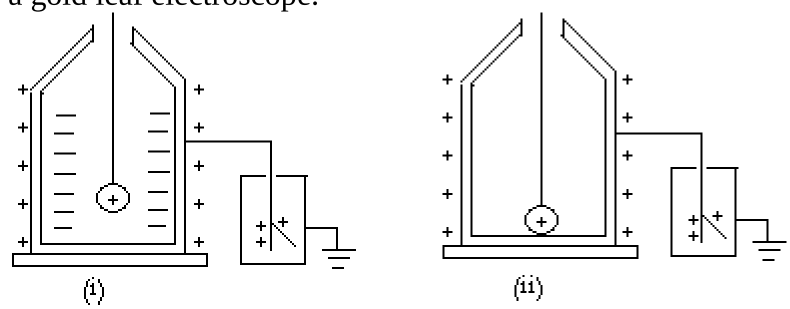
This is an example of an electrostatic generator which separates charges by induction and thus building up very great charges and potential difference.

A Van Da Graaff generator has a hollow metal sphere supported by an insulating stand, silk belt running over pulleys driven by a motor and two electrodes $E_1$ and $E_2$ as shown in the diagram below.



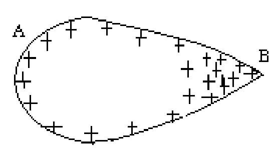

Electrode $E_1$is made about 1000v +ve with respect to the earth by a battery. The high electric field at sharps of $E_1$ ionize the air molecules there and the positive charges are repelled to the belt, which carries them up into the sphere. The positive charge on the belt induces a negative charge on sharp points of electrode $E_2$ and positive charge on the sphere on which the blunt end of $E_2$ is connected.  The high electric field at $E_2$ ionize the air around and the negative ions are repelled to the belt, hence discharging it. In this way the sphere gradually charges positively until its potential is about million volts relative to the earth.

**Ice pail experiment.**
This experiment was made by Farady using an ice pail. It involves a pail placed on an insulator and connected to a gold leaf electroscope.



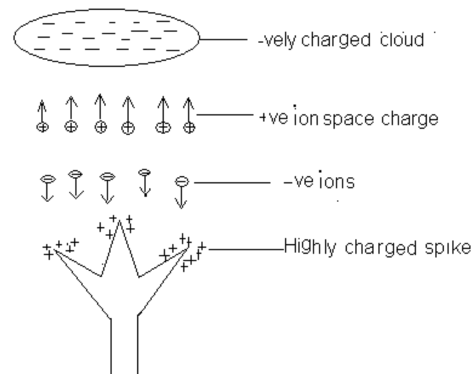
A positively charged metal sphere is lowered using a thread into a small can connected to a gold leaf electroscope. The leaf of the electroscope diverges as shown in figure (i). The sphere is withdrawn from the can and the leaf of the electroscope is seen to collapse.
When the sphere is lowered again into the can, the leaf diverges by the same amount as before. The sphere is then made to touch the inner surface of the can as shown in figure (ii) and then withdrawn. The divergence remains the same. When the sphere is now tested for charge, it is found to have no charge. Hence there is no charge inside the can but since the electroscope leaf remains diverged, charge exists inside the can.

**Charge distributions on surfaces of conductors.**
When different shapes of conductors are investigated using a proof plane, it is found that the charge per unit area or surface density increases with the curvature of the conductor as shown below.




The high concentration of charge at the sharp point B leads to a high charge density at that point. As a result, the air molecules around B are ionized. The ions with the same sign of charge are repelled away while those with those with opposite sign are attracted and hence there is a reduction of charge at B due to neutralization. This process is called **corona discharge**.

**Lightening conductor.**
This protects a house from being destroyed by lightening. Its comprised of a sharp pointed metal rod fixed at the highest point of the building and connected with a thick copper plate buried into the ground.




When a negatively charged cloud passes over a lightening conductor, positive charges are induced at the spikes and the negative charges are conducted into the ground through the copper plate. Because of the high density of +ve charges at the spikes, the air molecules are ionized.